Oscillations Practice Quiz

Name: _____ **Date:** _____

$g = 10$ m/s^2

Part A. Multiple Choice

1. A cart of mass 0.80 kg rides on a level air track tied to a spring, and $x(t) = (0.060\text{ m})\cos\big((5.0\text{ rad/s})t\big)$. The spring constant is

A. 0.032 N/m
B. 20 N/m
C. 4.0 N/m
D. 31 N/m

2. Two blocks of equal mass oscillate on identical springs, one described by $x = A\cos\omega t$ and the other by $x = A\cos(\omega t + \pi/2)$. Which statement describes them at $t = 0$?

A. the second has the longer period, because its argument is larger at every instant
B. the second has the smaller amplitude, because the phase constant reduces the cosine
C. the second is not in simple harmonic motion, because the phase constant breaks the differential equation
D. the first is at its positive end and the second is at the equilibrium position, and the two have the same period

3. A block has displacement $x(t) = (0.10\text{ m})\cos\big((6.0\text{ rad/s})t + \pi/4\big)$. The first time after $t = 0$ at which it passes through the equilibrium position is

A. 0.13 s
B. 0.26 s
C. 0.065 s
D. 0.39 s

4. A block of mass 0.60 kg hangs from two springs joined end to end, of spring constants 30 N/m and 60 N/m. Its period is

A. 0.89 s
B. 0.63 s
C. 1.1 s
D. 0.51 s

5. A block hangs from a spring and oscillates with period T. The spring is cut exactly in half and the same block hangs from one of the halves. The new period is

A. $2T$
B. $T/\sqrt{2}$
C. $T/2$
D. $T\sqrt{2}$

6. A block on a spring oscillates with angular frequency ω. The block is replaced by one of four times the mass, and the spring by one of twice the spring constant. The new angular frequency is

A. $\omega/2$
B. $\omega\sqrt{2}$
C. 2ω
D. $\omega/\sqrt{2}$

7. The velocity of a block in simple harmonic motion is $\dfrac{dx}{dt} = (0.48\text{ m/s})\cos\big((4.0\text{ rad/s})t\big)$. The greatest magnitude of its acceleration is

A. 0.48 m/s^2
B. 7.7 m/s^2
C. 1.9 m/s^2
D. 0.12 m/s^2

8. A block in simple harmonic motion has $x(t) = (0.050\text{ m})\sin\big((10\text{ rad/s})t\big)$. The first time after $t = 0$ at which its kinetic energy equals the energy stored in the spring is

A. 0.079 s
B. 0.16 s
C. 0.039 s
D. 0.052 s

9. For an oscillating block, $\dfrac{d^2x}{dt^2}$ is plotted against x and the points lie on a straight line through the origin. The slope of that line

A. cannot be constant, because the acceleration depends on the square of the displacement
B. is $-\omega^2$, so the period follows from the graph with no timing at all
C. is $-k$, so the spring constant follows from the graph without knowing the mass
D. is the greatest magnitude of the acceleration, reached at the ends of the travel

10. A block of mass 0.75 kg oscillates on a spring of spring constant 24 N/m with an amplitude of 0.25 m. Its greatest speed is

A. 8.0 m/s
B. 0.044 m/s
C. 0.71 m/s
D. 1.4 m/s

11. A particle of mass m moves in one dimension in the potential energy $U(x) = bx^4$, with b a positive constant, and oscillates about the origin. The motion is

A. periodic but not simple harmonic, and its period depends on the amplitude
B. simple harmonic with angular frequency $\sqrt{4b/m}$
C. periodic and simple harmonic, with a period that does not depend on the amplitude
D. not periodic, because the restoring force is not proportional to the displacement

12. A uniform rod of length L swings about a horizontal axle through one end. A simple pendulum has a string of the same length L. For small swings,

A. the two have the same period, because they have the same length
B. the comparison depends on the mass of the rod, which does not divide out of a physical pendulum
C. the rod has the shorter period, because its effective length is $2L/3$
D. the simple pendulum has the shorter period, because all of its mass is at the far end

13. A uniform disk of radius 0.20 m hangs from a horizontal axle through a point on its rim and swings in its own plane through a small angle. Its period is

A. 1.3 s
B. 1.1 s
C. 0.63 s
D. 0.89 s

14.* A block on a frictionless plane inclined at angle θ is attached to a spring that lies along the plane. Compared with the same block and spring on a level surface, the period of its oscillation is

A. longer by a factor of $1/\sqrt{\cos\theta}$, because the plane reduces the restoring force
B. shorter, because the component of the weight along the plane adds to the spring force on the way down
C. the same only for small θ, since a steep plane makes the motion other than simple harmonic
D. the same, because the component of the weight along the plane is constant and only moves the equilibrium position

15.* A 0.50 kg block on a spring of constant 50 N/m slides on a surface with coefficient of kinetic friction 0.20. It is released from rest 0.30 m from equilibrium. When it next comes to rest, its distance from equilibrium is

A. 0.22 m
B. 0.30 m
C. 0.26 m
D. 0.28 m

16.* For a block in simple harmonic motion of amplitude A and angular frequency ω, the velocity $\dfrac{dx}{dt}$ is plotted against the displacement x. The curve traced out over one period is

A. an ellipse, traced once, with semi-axes A and $A\omega$
B. a circle of radius A, traced once
C. a straight line through the origin, retraced in both directions
D. a parabola, because the kinetic energy depends on the square of the velocity

17.* A particle of mass 0.50 kg moves in the potential energy $U(x) = (12\ \text{J/m}^2)x^2$. At the instant it is at $x = 0.30$ m it is moving at 2.0 m/s. The amplitude of its motion is

A. 0.17 m
B. 0.42 m
C. 0.30 m
D. 0.59 m

18.* A uniform rod of length L is pivoted at a point a distance d from its center and swings as a physical pendulum through small angles. As d is increased from near zero toward $L/2$, the period

A. rises steadily, because the rotational inertia about the pivot grows
B. does not change, because the mass of the rod divides out of the period
C. falls to a least value and then rises again
D. falls steadily, because the weight acts farther from the pivot

* Challenging.

Part B. Reasoning

1. Explain why $x = A\cos(\omega t + \phi)$ and $x = B\sin\omega t + C\cos\omega t$ describe the same family of motions, and state what the two constants in each form are fixed by.

2. A student writes that the acceleration of a block on a spring is constant, because the spring constant and the mass are both constant. Explain the error, using the equation of motion.

3. A disk hangs from a wire that resists twisting with a torque $\tau = -\kappa\theta$. Explain why the disk twists back and forth in simple harmonic motion, and give its angular frequency.

4. For $x(t) = A\cos\omega t$, explain why each differentiation with respect to time multiplies the amplitude by ω and advances the curve by a quarter of a period.

5. Two blocks obey the same equation $d^2x/dt^2 = -\omega^2 x$ but their recorded graphs have different heights. Explain why the graphs must still have the same width, and what fixes the height.

6. Explain how $U(x) = \frac{1}{2}kx^2$ follows from $F = -kx$ rather than being quoted, and state what the constant of integration is chosen to be and why that is a choice.

7. Explain why the period of a physical pendulum does not depend on its mass, even though the mass appears both in its rotational inertia and in the torque of its weight.

8.* Explain how $x = A\cos(\omega t + \phi)$ can be shown to solve $d^2x/dt^2 = -\omega^2 x$ without solving the equation, and state what that check does and does not establish.

9.* One block and two identical springs can be arranged with the springs side by side or end to end. Explain which arrangement gives the longer period, arguing from the stretches rather than quoting a result.

10.* A motion sensor records the displacement of a block on a spring against time. Explain why that record alone cannot give both the mass and the spring constant, and how one known mass makes both of them available.

11.* A particle of mass m moves in the potential energy $U(x) = \frac{1}{2}kx^2 + bx$, with b a constant. Explain why the motion is still simple harmonic, give its angular frequency, and state where the motion is centered.

12.* Explain what is meant by the center of oscillation of a physical pendulum, state where it lies for a uniform rod pivoted at one end, and explain what happens if the rod is pivoted there instead.

* Challenging.

Part C. Problems

1. A block of mass 0.40 kg on a frictionless horizontal surface is attached to a spring of spring constant 90 N/m.

(a) Calculate the angular frequency of the oscillation.

(b) Calculate the period of the oscillation.

2. A block oscillates with $x(t) = (0.080 \text{ m}) \cos\big((12 \text{ rad/s})t\big)$.

(a) Calculate $\dfrac{dx}{dt}$ at $t = 0.10$ s.

(b) Calculate $\dfrac{d^2x}{dt^2}$ at $t = 0.10$ s.

3. A block of mass 0.50 kg is to be hung from two springs of spring constants 20 N/m and 30 N/m.

(a) Calculate the period when the two springs are side by side, both carrying the block.

(b) Calculate the period when the two springs are joined end to end, with the block hanging below.

4. A block hangs from a vertical spring. It is pulled 0.060 m below its resting position and released, and it first returns to that same point 0.90 s later.

(a) Calculate the angular frequency of the oscillation.

(b) Calculate the greatest magnitude of the acceleration of the block.

5. A block moves with $x(t) = (0.060 \text{ m}) \sin\big((8.0 \text{ rad/s})t + \pi/3\big)$.

(a) Calculate the greatest speed of the block.

(b) Calculate the first time after $t = 0$ at which the block is at the equilibrium position.

6. A particle of mass 0.50 kg moves in the potential energy $U(x) = (32 \text{ J/m}^2)x^2$ and oscillates with an amplitude of 0.15 m.

(a) Calculate the spring constant of the force acting on the particle.

(b) Calculate the greatest speed of the particle.

7. A thin uniform hoop of mass 0.80 kg and radius 0.30 m hangs from a nail in a wall and swings in its own plane through a small angle. About its center, $I_{\text{cm}} = MR^2$.

(a) Calculate the rotational inertia of the hoop about the nail.

(b) Calculate the period of the hoop for small swings.

8.* A block of mass 0.30 kg moves so that $\dfrac{d^2x}{dt^2} = -(200\ \text{s}^{-2})x$. At $t = 0$ it is at $x = 0.050$ m and moving at $+0.40$ m/s.

(a) Calculate the amplitude of the motion.

(b) Calculate the phase constant ϕ in $x = A\cos(\omega t + \phi)$.

9.* A block of mass 0.40 kg hangs from a spring of spring constant 60 N/m fixed to the ceiling. A second spring of spring constant 30 N/m is then attached below the block and anchored to the floor, both springs vertical and both taut.

(a) Calculate the effective spring constant of the pair for vertical motion of the block.

(b) Calculate the period of the oscillation.

10.* A block oscillates with an amplitude of 0.10 m and an angular frequency of 5.0 rad/s, starting from rest at $x = +A$ at $t = 0$.

(a) Calculate the first time at which the magnitude of the acceleration is half its greatest value.

(b) Calculate the speed of the block at that instant.

11.* A particle of mass 0.20 kg moves in the potential energy $U(x) = (40\ \text{J/m}^2)x^2 + (500\ \text{J/m}^4)x^4$ and is released from rest at $x = 0.10$ m.

(a) Calculate the total mechanical energy of the particle.

(b) Calculate the speed of the particle at $x = 0.050$ m.

12.* A uniform meter stick of length 1.00 m swings in a vertical plane about a frictionless horizontal axle through a point 0.25 m from one end. About its center, $I_{\text{cm}} = \frac{1}{12}ML^2$.

(a) Calculate the rotational inertia of the meter stick about the axle, in terms of its mass M.

(b) Calculate the period of the meter stick for small swings.

* Challenging.

AP Physics C: Mechanics, Equation Sheet

The constants, conventions and equations on this sheet are those the College Board provides on the AP Physics exam reference information.

CONSTANTS AND CONVERSION FACTORS

$G = 6.67 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)} = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

universal gravitational constant

$g = 9.8\ \mathrm{m/s^2}$

magnitude of the acceleration due to gravity at the surface of the Earth

$g = 9.8\ \mathrm{N/kg}$

magnitude of the gravitational field strength at the surface of the Earth

CONVENTIONS

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT SYMBOLS

hertz	Hz
newton	N
joule	J
second	s
kilogram	kg
watt	W
meter	m

TRIGONOMETRIC FUNCTIONS AT COMMON ANGLES

θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular solid

$V = \ell w h$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right triangle

$a^2 + b^2 = c^2$

$\sin\theta = a/c$

$\cos\theta = b/c$

$\tan\theta = a/b$

KINEMATICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x\,(x - x_0)$$

$$\Delta x = \int v_x(t)\,dt$$

$$\Delta v_x = \int a_x(t)\,dt$$

CENTER OF MASS

$$\vec{x}_{\mathrm{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\vec{r}_{\mathrm{cm}} = \frac{\int \vec{r}\,dm}{\int dm}$$

$$\lambda = \frac{d}{d\ell}m(\ell)$$

$$\vec{v}_{\mathrm{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

FORCE AND TRANSLATIONAL DYNAMICS

$$\vec{a}_{\mathrm{sys}} = \frac{\sum \vec{F}}{m_{\mathrm{sys}}} = \frac{\vec{F}_{\mathrm{net}}}{m_{\mathrm{sys}}}$$

$$\vec{F}_{\mathrm{net}} = \frac{d\vec{p}}{dt}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\left|\vec{F}_f\right| \le \left|\mu \vec{F}_N\right|$$

$$\vec{F}_s = -k\Delta\vec{x}$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$T = \frac{1}{f}$$

WORK, ENERGY AND POWER

$$K = \tfrac{1}{2}mv^2$$

$$W = \int_a^b \vec{F}\cdot d\vec{r}$$

$$\Delta K = \sum W_i = \sum F_{\parallel,i}\,d_i$$

$$\Delta U = -\int_a^b \vec{F}_{\mathrm{cf}}(r)\cdot d\vec{r}$$

$$F_x = -\frac{dU(x)}{dx}$$
$$U_s = \tfrac{1}{2}k\left(\Delta x\right)^2$$
$$U_G = -G\frac{m_1 m_2}{r}$$
$$\Delta U_g = mg\Delta y$$
$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$
$$P_{\text{inst}} = \frac{dW}{dt}$$

LINEAR MOMENTUM

$$\vec{p} = m\vec{v}$$
$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t)\,dt = \Delta\vec{p}$$

TORQUE AND ROTATIONAL DYNAMICS

$$\omega = \frac{d\theta}{dt}$$
$$\alpha = \frac{d\omega}{dt}$$
$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right)$$
$$v = r\omega$$
$$a_T = r\alpha$$
$$\vec{\tau} = \vec{r}\times\vec{F}$$
$$I_{\text{tot}} = \sum I_i = \sum m_i r_i^2$$
$$I = \int r^2\,dm$$
$$I' = I_{\text{cm}} + Md^2$$
$$\alpha_{\text{sys}} = \frac{\Sigma\tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

ENERGY AND MOMENTUM OF ROTATING SYSTEMS

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2$$
$$W = \int \tau\cdot d\theta$$
$$\vec{L} = \vec{r}\times\vec{p} = I\vec{\omega}$$
$$\Delta L = \int \tau\,dt$$
$$\Delta x_{\text{cm}} = r\Delta\theta$$

OSCILLATIONS

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$
$$T_s = 2\pi\sqrt{\frac{m}{k}}$$
$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$
$$T_{\text{phys}} = 2\pi\sqrt{\frac{I}{mgd}}$$
$$x = x_{\max}\cos\left(\omega t + \phi\right)$$

VECTORS

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$
$$\vec{r} = \left(A\hat{\imath} + B\hat{\jmath} + C\hat{k}\right)$$
$$\vec{C} = \vec{A} + \vec{B}$$
$$\vec{C} = \left(A_x + B_x\right)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath}$$

CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$
$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$
$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$
$$\frac{d}{dx}\left(\ln ax\right) = \frac{1}{x}$$
$$\frac{d}{dx}\left[\sin(ax)\right] = a\cos(ax)$$
$$\frac{d}{dx}\left[\cos(ax)\right] = -a\sin(ax)$$
$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$
$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$
$$\int \frac{dx}{x+a} = \ln\left|x+a\right|$$
$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$
$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

IDENTITIES

$$\log\left(a\cdot b^x\right) = \log a + x\log b$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin\left(2\theta\right) = 2\sin\theta\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

WHAT THE LETTERS MEAN

a	acceleration
d	distance
E	energy
f	frequency
F	force
I	rotational inertia
J	impulse
k	spring constant
K	kinetic energy
ℓ	length
L	angular momentum
m	mass
M	mass
p	momentum
P	power
r	radius, distance, or position
t	time
T	period
U	potential energy
v	velocity or speed
W	work
x	position or distance
y	vertical position
α	angular acceleration
θ	angular position
λ	linear mass density
μ	coefficient of friction
τ	torque
ϕ	phase angle
ω	angular frequency or angular speed