Energy and Momentum of Rotating Systems Practice Quiz

Name: _____ **Date:** _____

$g = 10$ m/s^2

Part A. Multiple Choice

1. A stack of platters inside a drive has a rotational inertia of 8.0×10^{-5} kg·m^2 and turns at 600 rad/s. Its rotational kinetic energy is

A. 0.048 J
B. 14.4 J
C. 28.8 J
D. 0.024 J

2. A flywheel has a rotational inertia of 0.50 kg·m^2. The angular velocity at which it holds 400 J of rotational kinetic energy is

A. 1600 rad/s
B. 28 rad/s
C. 20 rad/s
D. 40 rad/s

3. A merry-go-round turns at a steady rate. One child sits halfway from the axle to the rim, and a second child of equal mass sits at the rim. Compared with the first child, the kinetic energy of the second child is

A. four times as great, since the speed of a rider is proportional to that distance
B. twice as great, because the distance from the axle is twice as great
C. the same, because both children turn at the same angular velocity
D. eight times as great, because the kinetic energy depends on the cube of the distance

4. A 1.5 kg bucket hangs from a cord wrapped around a solid uniform cylinder of mass 2.0 kg and radius 0.12 m on a fixed axle. The bucket is released from rest and falls 1.2 m. Its speed there is

A. 3.2 m/s
B. 6.0 m/s
C. 3.8 m/s
D. 4.9 m/s

5. A thin ring, a uniform disk, and a uniform solid sphere have the same mass and the same outer radius, and each turns about its central axis at the same angular velocity. Their rotational kinetic energies, from greatest to least, are

A. all three are equal
B. ring, disk, sphere
C. sphere, disk, ring
D. disk, ring, sphere

6. On a graph of torque against angular position, the area between the curve and the horizontal axis gives

A. the change in the angular momentum
B. the average torque over that interval
C. the angular impulse delivered by that torque
D. the work done by that torque

7. A torque on a wheel falls in a straight line from 12 N·m at $\theta = 0$ to zero at $\theta = 6.0$ rad. The work this torque does over that interval is

A. 18 J
B. 2.0 J

C. 36 J
D. 72 J

8. A fan turns at a steady angular velocity while a motor torque drives it and a friction torque acts against it. Over any interval, the net work done on the fan is
A. zero, since the rotational kinetic energy does not change
B. positive, because the motor is the stronger of the two torques
C. negative, because friction always removes energy
D. zero, because the two torques act at the same distance from the axle

9. A potter's wheel with $I = 0.90$ kg·m^2 turns at 14 rad/s. A tool pressed against the rim brings it to rest after 35 rad. The magnitude of the friction torque is
A. 0.36 N·m
B. 2.5 N·m
C. 5.0 N·m
D. 0.18 N·m

10. Two wheels start from rest. The same net torque acts on each one through the same angle, and the second wheel has twice the rotational inertia of the first. At the end of that angle, the two wheels have
A. equal angular velocities, and the second wheel has more kinetic energy
B. equal kinetic energies and equal angular velocities
C. the second wheel has twice the kinetic energy and half the angular velocity
D. equal kinetic energies, and the second wheel turns more slowly

11. A 0.40 kg dart flies at 15 m/s along a straight horizontal path. The path passes 0.60 m from a point P. The magnitude of the dart's angular momentum about P is
A. 3.6 kg·m^2/s
B. 6.0 kg·m^2/s
C. 9.0 kg·m^2/s
D. 0.24 kg·m^2/s

12. A sled slides in a straight line at a constant velocity across level ice. The point Q lies on the ice 3 m to one side of its path. As the sled travels, its angular momentum about Q is
A. increasing, because the straight-line distance from Q to the sled grows steadily
B. zero except at the instant the sled is closest to Q
C. constant and not zero, since the perpendicular distance from Q to the path is fixed
D. zero at every instant, because the sled does not turn

13. A flywheel with $I = 0.25$ kg·m^2 turns at 40 rad/s. A steady torque of 2.0 N·m acts against the rotation for 3.0 s. The magnitude of its angular momentum is then
A. 8.5 kg·m^2/s
B. 4.0 kg·m^2/s
C. 16 kg·m^2/s
D. 6.0 kg·m^2/s

14. A turntable turns at a steady rate, so its angular momentum about the axle is constant. Over that interval, the net torque on the turntable is
A. constant and not zero, because the turntable keeps turning
B. zero only if no single torque at all acts on the turntable
C. equal to $I\omega$ divided by the time
D. zero, because the angular momentum does not change

15. A wheel starts from rest and reaches a given angular velocity under a large torque acting for a short time. An identical wheel reaches the same angular velocity under a small torque acting for a long time. The two wheels received

A. equal net work, and a greater angular impulse on the first wheel
B. a greater angular impulse and more net work on the first wheel
C. equal angular impulses and equal net work
D. equal angular impulses, and more net work on the first wheel

16. A space station turns slowly with no outside torque on it. Crew members then walk from the hub out to the rim. The total angular momentum of the station and crew

A. stays the same, and the rate of turning decreases
B. stays the same, and the rate of turning increases
C. increases, because the crew moves farther from the axis
D. decreases, because the rotational inertia increases

17. A turning platform carries a folding arm, with no outside torque on it. With the arm out the system has $I = 0.90$ kg·m^2 at 1.2 rad/s. The arm folds in until $I = 0.30$ kg·m^2. The angular velocity is then

A. 1.2 rad/s
B. 3.6 rad/s
C. 0.40 rad/s
D. 2.1 rad/s

18. A spinning disk and a second disk at rest sit on a common axle. They are pressed together until they turn as one. Compared with their values before, the total angular momentum and the total kinetic energy after are

A. less angular momentum and less kinetic energy
B. the same kinetic energy and less angular momentum
C. the same angular momentum and less kinetic energy
D. the same angular momentum and the same kinetic energy

19. A turntable with $I = 0.020$ kg·m^2 turns freely at 4.0 rad/s. A 0.40 kg block is lowered straight down onto it and sticks 0.20 m from the axle. The angular velocity is then

A. 4.0 rad/s
B. 7.2 rad/s
C. 0.80 rad/s
D. 2.2 rad/s

20. A satellite spins at 3.0 rad/s with its booms folded, giving $I = 1.5$ kg·m^2. No outside torque acts about the spin axis. The booms extend until $I = 4.5$ kg·m^2. The work done on the satellite is

A. -4.5 J
B. 0 J
C. 4.5 J
D. 13.5 J

21. A wheel rolls without slipping along a level path. At any instant, the velocity of the part of the wheel in contact with the path is

A. directed backward with the speed of the center
B. zero
C. equal to the velocity of the center
D. twice the velocity of the center

22. A lawn roller of radius 0.30 m rolls without slipping and speeds up steadily from rest to 2.4 m/s in 4.0 s. Its angular acceleration is

A. 0.18 rad/s^2
B. 8.0 rad/s^2
C. 2.0 rad/s^2
D. 0.60 rad/s^2

23. A block slides from rest down a frictionless ramp. A uniform disk rolls without slipping from rest down an identical ramp, starting at the same height. At the foot of the ramp, the center of the disk moves
A. at the same speed as the block, because both started at the same height
B. faster than the block, because the disk also turns
C. more slowly than the block, because static friction removed energy from the disk
D. more slowly than the block, because part of the energy of the disk goes into rotation

24. A uniform solid sphere is released from rest and rolls without slipping down a slope. Its center drops 1.4 m by the foot of the slope. The speed of the center there is
A. 6.3 m/s
B. 4.5 m/s
C. 5.3 m/s
D. 3.7 m/s

25. A wheel is spun rapidly about its axle and then set down on a rough level floor with its center at rest. While it slips, the friction from the floor
A. acts forward, so the center of the wheel speeds up while the spin slows down
B. acts backward, so both the center and the spin slow down
C. acts forward, so the center and the spin both speed up
D. does no work on the wheel, because the contact point stays at rest

26. Two satellites of different masses travel in circular orbits of the same radius about a planet. Their orbital speeds are
A. greater for the heavier satellite, because gravity pulls on it harder
B. greater for the lighter satellite, because less force is required
C. different, and the ratio depends on the mass of the planet
D. equal, because the mass of the satellite cancels

27. A moon of mass 1.2×10^{23} kg has a small satellite in a circular orbit at 1.0×10^{3} m/s. With $G = 6.67 \times 10^{-11}$ N·m^2/kg^2, the radius of that orbit is
A. 1.25×10^{-7} m
B. 1.6×10^{7} m
C. 8.0×10^{6} m
D. 8.0×10^{9} m

28. A moon has a mass of 7.3×10^{22} kg and a radius of 1.7×10^{6} m. With $G = 6.67 \times 10^{-11}$ N·m^2/kg^2, the escape speed from its surface is
A. 2.4×10^{3} m/s
B. 1.7×10^{3} m/s
C. 5.7×10^{6} m/s
D. 1.8 m/s

29. A 1.5×10^{3} kg probe passes the closest point of its orbit 6.0×10^{6} m from a planet's center at 9.0×10^{3} m/s. At the farthest point it is 2.4×10^{7} m from the center. Its kinetic energy there is
A. 1.7×10^{6} J
B. 3.8×10^{9} J
C. 6.1×10^{10} J

D. 9.7×10^{11} J

30. Two satellites of equal mass travel in circular orbits about the same planet, one at three times the radius of the other. Compared with the inner satellite, the outer satellite has

A. less kinetic energy and a total energy that is more negative

B. more kinetic energy and a total energy that is less negative

C. the same total energy, because gravity does no work on either one

D. less kinetic energy and a total energy that is nearer zero, so less negative

31.* A 3.0 kg block on a frictionless table is joined by a cord over a pulley to a 2.0 kg hanging block. The pulley is a uniform disk of mass 4.0 kg and the cord does not slip. The hanging block falls 0.80 m from rest. Its speed is then

A. 1.9 m/s

B. 4.0 m/s

C. 2.1 m/s

D. 2.5 m/s

32.* A door panel is uniform, 30 kg and 1.0 m wide, hinged along one edge, so $I = \frac{1}{3}ML^2$. From rest, a 40 N force acts at the outer edge for 1.5 s, at 30° to the line from the hinges. Then ω is

A. 3.0 rad/s

B. 6.0 rad/s

C. 5.2 rad/s

D. 1.0 rad/s

33.* On one axle, disk P has $I = 0.40$ kg·m^2 and turns counterclockwise at 9.0 rad/s, while disk Q has $I = 0.20$ kg·m^2 and turns clockwise at 6.0 rad/s. Pressed together, they turn as one. The energy converted is

A. 4.8 J

B. 15.0 J

C. 0.60 J

D. 19.8 J

34.* A ball rolls without slipping along level ground and then runs up a ramp so smooth that friction on it can be ignored. The greatest height the ball reaches is

A. the same as on a rough ramp, because mechanical energy is conserved in both cases

B. greater than on a rough ramp, because no friction force removes energy from the ball

C. greater than for a block of equal mass sliding up at the same speed, since the ball carries rotational energy as well

D. less than on a rough ramp, since its spin does not slow and that energy stays unused

35.* A spacecraft moves in an elliptical orbit about a planet, for which $GM = 4.0 \times 10^{14}$ m^3/s^2. Its nearest point is 4.0×10^6 m from the center and its farthest is 1.2×10^7 m. Its speed at the nearest point is

A. 1.4×10^4 m/s

B. 8.7×10^3 m/s

C. 1.2×10^4 m/s

D. 1.0×10^4 m/s

* Challenging.

Part B. Reasoning

1. A potter places a lump of clay near the axis of a wheel, then repeats the trial with the same lump near the outer edge. The wheel reaches the same angular velocity each time. Explain which placement gives the clay more kinetic energy.

2. A helicopter hovers at a fixed height, so its center of mass stays in one place while the rotor turns at full speed. Explain why the helicopter and its rotor together still have kinetic energy.

3. A thrown football moves down the field and spins about its long axis at the same time. Explain why its total kinetic energy has to be written as two terms, and state what each term depends on.

4. A drone hovers with four rotors turning at the same rate, two of them clockwise and two counterclockwise. Explain why the total rotational kinetic energy of the four rotors is not zero.

5. A sailor leans hard on the handle of a capstan, and the capstan does not turn. Explain whether the sailor does any work on the capstan, and state what would have to be true for that work to be above zero.

6. As a drawbridge is raised, the torque of gravity about its hinge gets weaker as the bridge turns. Explain why $W = \tau\,\Delta\theta$ cannot be used for the work gravity does here, and state what does give that work.

7. A hand drill is turned by a crank. In one trial the hand grips the end of the handle, and in another it grips the middle. The same force turns the drill through the same angle both times. Explain in which trial the hand does more work.

8. A bird flies in a straight horizontal line at a steady speed, passing to one side of a lighthouse. Explain why the bird has angular momentum about the base of the lighthouse, and why that angular momentum does not change as it flies.

9. A graph of the angular momentum of a turbine against time has a flat section, and the turbine keeps turning all through that section. Explain what the flat section tells you about the net torque on the turbine.

10. Two people push a revolving door with the same force for the same length of time. One pushes close to the central post, and the other pushes near the outer edge. Explain which push changes the angular momentum of the door more.

11. A diver leaves the board turning slowly with the body stretched out, then pulls into a tight tuck and turns much faster. Explain why the rotation speeds up while the diver is in the air.

12. A lump of putty is dropped onto the rim of a turntable that is spinning freely, and the putty sticks there. Explain why the angular momentum about the axle is the same before and after, while the kinetic energy is not.

13. The engine of a helicopter turns the main rotor one way. Explain why the body of the helicopter would turn the other way without a tail rotor, and explain what the tail rotor does about that.

14. Two skaters of equal mass glide along parallel lines in opposite directions, pass each other, and grab hands. They then turn together about the point midway between them. Explain where that rotation comes from.

15. A bicycle rolls along the road without slipping. Explain why the top of the tire moves forward at twice the speed of the hub, and state the speed of the part of the tire that touches the road.

16. A truck speeds up on dry pavement without its wheels slipping, and later the driver locks the brakes and the truck skids. Explain why the friction at the road is static in the first case and kinetic in the second.

17. A wheel rolls without slipping along a level floor and then crosses a patch of ice where the friction is negligible. Explain what happens to the speed of its center and to its angular velocity while it is on the ice.

18. A satellite travels in a circular orbit around a planet. Explain why the total mechanical energy of the satellite and planet system is below zero, and explain what a total of exactly zero would tell you.

19. A comet moves on a long elliptical orbit around the Sun. Explain why it spends most of each orbit far from the Sun and only a short part of each orbit near the Sun.

20. Two satellites travel in circular orbits around the same planet, one at a small radius and one at a large radius. Explain which satellite completes one orbit in less time.

21.* A hoop and a uniform disk of equal mass and radius turn on separate fixed axles. The same work is done on each from rest, so each ends with the same rotational kinetic energy. Explain which one then has the greater angular momentum.

22.* A block hangs from a cord wrapped around a pulley that has mass, and the block is then released. Explain why the tension in the cord is below the weight of the block, and explain what the cord does to the energy of each of the two.

23.* A student sits at rest on a stool that turns freely, holding a spinning bicycle wheel with its axis vertical. The student then turns the wheel over, so that its axis is vertical the other way. Explain why the student begins to turn.

24.* A uniform solid sphere and a hoop of the same mass roll without slipping along a level floor with the same total kinetic energy. Each then rolls up a ramp. Explain which one rises higher before it stops.

25.* A satellite in a circular orbit fires its engine briefly in the direction it is moving, and it ends up in a new circular orbit at a greater radius. Explain why its speed in the new orbit is below its speed in the old one.

* Challenging.

Part C. Problems

1. A kiln turntable is a uniform disk of mass 24 kg and radius 0.50 m. Two fired bricks, each of mass 3.0 kg, sit on the rim on opposite sides. The loaded turntable turns about its central axis at 8.0 rad/s.

(a) Calculate the rotational inertia of the loaded turntable about its central axis.

(b) Calculate the rotational kinetic energy of the loaded turntable.

2. A dock worker turns a capstan drum by pushing a handle along the tangent with a steady force of 120 N at 0.75 m from the axis. The drum has rotational inertia 18 kg·m^2, starts from rest, and turns through 6.0 rad on a frictionless axle.

(a) Calculate the work the worker does on the drum.

(b) Calculate the angular velocity of the drum at the end of the 6.0 rad.

3. A potter's wheel has rotational inertia 0.90 kg·m^2 and turns at 14 rad/s. The potter presses a rib against the rim, and the rib exerts a steady torque of 1.2 N·m against the rotation for 5.0 s.

(a) Calculate the angular momentum of the wheel before the rib touches it.

(b) Calculate the angular velocity of the wheel at the end of the 5.0 s.

4. A spinning satellite has rotational inertia 100 kg·m^2 about its spin axis and turns at 3.0 rad/s. Two antenna booms are extended, and the rotational inertia about that axis becomes 250 kg·m^2. No external torque acts about the spin axis.

(a) Calculate the angular velocity of the satellite with the booms extended.

(b) Calculate the change in the rotational kinetic energy of the satellite.

5. A uniform solid cylinder of mass 4.0 kg and radius 0.12 m rolls without slipping along a level floor, and its total kinetic energy is 27 J.

(a) Calculate the speed of the center of the cylinder.

(b) Calculate the angular velocity of the cylinder.

6. A small moon travels in a circular orbit of radius 1.0×10^7 m about a planet of mass 6.0×10^{23} kg. Use $G = 6.67 \times 10^{-11}$ N·m^2/kg^2.

(a) Calculate the orbital speed of the moon.

(b) Calculate the period of the orbit.

7. A uniform beam of mass 1.2 kg and length 1.5 m turns in a vertical plane about a frictionless hinge at one end. The beam is held horizontal and released from rest. About the hinge, $I = \frac{1}{3}ML^2$.

(a) Calculate the kinetic energy of the beam when it reaches the vertical position.

(b) Calculate the angular velocity of the beam at the vertical position.

8. A motor is set so that the torque on a drill shaft rises in a straight line from zero at $\theta = 0$ to 12 N·m at $\theta = 4.0$ rad, and then holds at 12 N·m until $\theta = 9.0$ rad.

(a) Calculate the work done on the shaft from $\theta = 0$ to $\theta = 4.0$ rad.

(b) Calculate the total work done on the shaft from $\theta = 0$ to $\theta = 9.0$ rad.

9. A drive belt exerts a torque of 2.5 N·m on a pulley in the direction it turns for 4.0 s. A brake then exerts 6.0 N·m against the rotation for 1.5 s. The pulley has rotational inertia 0.80 kg·m^2 and starts from rest.

(a) Calculate the net angular impulse delivered to the pulley over the whole 5.5 s.

(b) Calculate the angular velocity of the pulley at the end of the 5.5 s.

10. A turntable with rotational inertia 0.50 kg·m^2 turns at 6.0 rad/s on a frictionless axle. A 2.0 kg block rests at the center, and it slides out to a radius of 0.40 m, where it stays.

(a) Calculate the rotational inertia of the turntable and block once the block is at 0.40 m.

(b) Calculate the angular velocity of the turntable and block once the block is at 0.40 m.

11. A bicycle wheel, modeled as a hoop of mass 1.8 kg and radius 0.33 m, rolls without slipping along level ground at 5.0 m/s and then rolls without slipping up a hill.

(a) Calculate the total kinetic energy of the wheel on the level ground.

(b) Calculate the height the wheel rises up the hill before it stops.

12. The escape speed from the surface of a moon is 2.4×10^3 m/s. A rover is launched straight up from that surface at 3.0×10^3 m/s, and gravity from the moon is the only force on it.

(a) Calculate the speed of the rover when it is very far from the moon.

(b) Calculate the fraction of the launch kinetic energy that the rover still has when it is very far from the moon.

13. A rotating drum is built from an inner hub, a uniform disk of mass 10 kg and radius 0.20 m, and an outer rim, a hoop of mass 5.0 kg and radius 0.50 m. The hub and the rim turn together about the same axis at 12 rad/s.

(a) Calculate the rotational inertia of the drum about that axis.

(b) Calculate the fraction of the rotational kinetic energy of the drum that is in the rim.

14. A flywheel with rotational inertia 2.5 kg·m^2 turns at 20 rad/s. The drive is disconnected, and a constant friction torque at the bearing slows the flywheel to 8.0 rad/s over 60 rad.

(a) Calculate the change in the rotational kinetic energy of the flywheel.

(b) Calculate the magnitude of the friction torque at the bearing.

15. A bicycle and its rider, 80 kg in all, travel in a straight line at a steady 8.0 m/s along a level path. A lamp post stands 6.0 m from the path.

(a) Calculate the magnitude of the angular momentum of the bicycle and rider about the lamp post.

(b) Calculate the distance from the lamp post to the rider at the instant when the angle between the line from the post to the rider and the velocity of the rider is 30°.

16. A uniform disk of mass 6.0 kg and radius 0.40 m turns freely about its central axis at 9.0 rad/s. A 1.5 kg lump of putty is dropped straight down onto the disk and sticks to it at the rim.

(a) Calculate the angular velocity of the disk and putty just after the putty sticks.

(b) Calculate the kinetic energy converted to internal energy while the putty and the disk slip against each other.

17. A 12 kg wheel, modeled as a uniform disk of radius 0.30 m, rolls without slipping along level ground at 6.0 m/s. It then coasts to a stop over 40 m.

(a) Calculate the total kinetic energy of the wheel at 6.0 m/s.

(b) Calculate the magnitude of the average resistive force acting on the wheel along its direction of motion.

18. A 900 kg weather satellite travels in a circular orbit of radius 1.5×10^7 m about a planet, and its orbital speed is 2.0×10^3 m/s.

(a) Calculate the gravitational potential energy of the satellite and planet system.

(b) Calculate the total mechanical energy of the satellite and planet system.

19. A fairground wheel carries six cars on a light frame, each car of mass 40 kg, spaced evenly around a circle of radius 10 m. The wheel turns about its center at 0.20 rad/s, and the rotational inertia of the frame itself is negligible.

(a) Calculate the rotational inertia of the loaded wheel about its center.

(b) Calculate the rotational kinetic energy of the loaded wheel.

20. A cable drum has rotational inertia 0.60 kg·m^2 and turns at 10 rad/s. Over the next 25 rad a motor exerts 2.7 N·m along the rotation while friction at the bearing exerts 1.2 N·m against it.

(a) Calculate the net work done on the drum over the 25 rad.

(b) Calculate the angular velocity of the drum at the end of the 25 rad.

21.* A uniform solid sphere of mass 2.0 kg and radius 0.10 m rolls without slipping along level ground at 7.0 m/s. It then runs up a ramp whose surface is so slick that friction on the ramp is negligible. For the sphere, $I = \frac{2}{5}MR^2$.

(a) Calculate the total kinetic energy of the sphere on the level ground.

(b) Calculate the greatest height the center of the sphere rises above its level on the ground.

22.* A uniform rod of mass 1.2 kg and length 1.0 m turns in a horizontal plane about a frictionless vertical axis through its center at 5.0 rad/s. A 0.30 kg beetle sits on the axis and then walks out to one end of the rod. For the rod about that axis, $I = \frac{1}{12}ML^2$.

(a) Calculate the angular velocity of the rod and beetle when the beetle reaches the end of the rod.

(b) Calculate the change in the rotational kinetic energy of the rod and beetle.

23.* A probe of mass 800 kg moves in a circular orbit of radius 8.0×10^6 m about a planet, where its speed is 3.0×10^3 m/s. A brief engine burn along the direction of motion raises its speed to 4.0×10^3 m/s, and the probe then follows an elliptical orbit. At the burn point and at the farthest point of the ellipse the velocity is perpendicular to the line to the center of the planet.

(a) Calculate the speed of the probe at the farthest point of the elliptical orbit.

(b) Calculate the distance from the center of the planet to the farthest point of the orbit.

24.* A uniform rod of mass 3.0 kg and length 1.2 m lies at rest on frictionless ice. A puck strikes one end of the rod and delivers an impulse of 6.0 N·s perpendicular to the rod at that end. About its center, the rod has $I = \frac{1}{12}ML^2$.

(a) Calculate the speed of the center of mass of the rod just after the impulse.

(b) Calculate the angular velocity of the rod about its center of mass just after the impulse.

25.* A playground turntable of radius 2.0 m has rotational inertia 200 kg·m^2 and turns on a frictionless axle. A 30 kg child stands on the rim, and the turntable and child are at rest. The child throws a 2.0 kg ball horizontally along the tangent at 8.0 m/s relative to the ground.

(a) Calculate the angular velocity of the turntable and child just after the throw.

(b) Calculate the kinetic energy of the turntable and child just after the throw.

* Challenging.

AP Physics 1, Equation Sheet

The constants, conventions and equations on this sheet are those the College Board provides on the AP Physics exam reference information.

CONSTANTS AND CONVERSION FACTORS

$G = 6.67 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)} = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

universal gravitational constant

$1\ \mathrm{atm} = 1.0 \times 10^5\ \mathrm{N/m^2} = 1.0 \times 10^5\ \mathrm{Pa}$

one atmosphere of pressure

$g = 9.8\ \mathrm{m/s^2}$

magnitude of the acceleration due to gravity at the surface of the Earth

$g = 9.8\ \mathrm{N/kg}$

magnitude of the gravitational field strength at the surface of the Earth

CONVENTIONS

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.
- Fluids are assumed to be ideal, and pipes are assumed to be completely filled by fluid, unless otherwise stated.

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT SYMBOLS

hertz	Hz
newton	N
joule	J
pascal	Pa
kilogram	kg
second	s
meter	m
watt	W

TRIGONOMETRIC FUNCTIONS AT COMMON ANGLES

θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular solid

$V = \ell w h$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right triangle

$a^2 + b^2 = c^2$

$\sin\theta = a/c$

$\cos\theta = b/c$

$\tan\theta = a/b$

KINEMATICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x\left(x - x_0\right)$

CENTER OF MASS

$$\vec{x}_{\mathrm{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\vec{v}_{\mathrm{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

FORCE AND TRANSLATIONAL DYNAMICS

$$\vec{a}_{\mathrm{sys}} = \frac{\sum \vec{F}}{m_{\mathrm{sys}}} = \frac{\vec{F}_{\mathrm{net}}}{m_{\mathrm{sys}}}$$

$$\vec{F}_{\mathrm{net}} = \frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\left|\vec{F}_f\right| \le \left|\mu\vec{F}_N\right|$$

$$\vec{F}_s = -k\Delta\vec{x}$$

$$a_c = \frac{v^2}{r}$$

WORK, ENERGY AND POWER

$$K = \frac{1}{2}mv^2$$

$$W = F_{\parallel}d = Fd\cos\theta$$

$$\Delta K = \sum W_i = \sum F_{\parallel,i}\, d_i$$

$$U_s = \frac{1}{2}k\left(\Delta x\right)^2$$

$$U_G = -\frac{Gm_1m_2}{r}$$
$$\Delta U_g = mg\Delta y$$
$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$
$$P_{\text{inst}} = F_{\parallel}v = Fv\cos\theta$$

LINEAR MOMENTUM

$$\vec{p} = m\vec{v}$$
$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$$

TORQUE AND ROTATIONAL DYNAMICS

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right)$$
$$v = r\omega$$
$$a_T = r\alpha$$
$$\tau = r_{\perp}F = rF\sin\theta$$
$$I = \sum m_i r_i^2$$
$$I' = I_{\text{cm}} + Md^2$$
$$\alpha_{\text{sys}} = \frac{\Sigma\tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

ENERGY AND MOMENTUM OF ROTATING SYSTEMS

$$K = \tfrac{1}{2}I\omega^2$$
$$W = \tau\Delta\theta$$
$$L = I\omega$$
$$L = rmv\sin\theta$$
$$\Delta L = \tau\Delta t$$
$$\Delta x_{\text{cm}} = r\Delta\theta$$

OSCILLATIONS

$$T = \frac{1}{f}$$
$$T_s = 2\pi\sqrt{\frac{m}{k}}$$
$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$
$$x = A\cos\left(2\pi ft\right)$$
$$x = A\sin\left(2\pi ft\right)$$

FLUIDS

$$\rho = \frac{m}{V}$$
$$P = \frac{F_{\perp}}{A}$$
$$P = P_0 + \rho gh$$
$$P_{\text{gauge}} = \rho gh$$
$$F_b = \rho Vg$$
$$A_1v_1 = A_2v_2$$
$$P_1 + \rho gy_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \tfrac{1}{2}\rho v_2^2$$

WHAT THE LETTERS MEAN

a	acceleration
A	amplitude or area
d	distance
E	energy
f	frequency
F	force
h	height
I	rotational inertia
J	impulse
k	spring constant
K	kinetic energy
ℓ	length
L	angular momentum
m	mass
M	mass
p	momentum
P	power or pressure
r	radius or distance
t	time
T	period
U	potential energy
v	velocity or speed
V	volume
W	work
x	position
y	vertical position
α	angular acceleration
θ	angle or angular position
μ	coefficient of friction
ρ	density
τ	torque
ω	angular speed