

Practice Test 2, Units 1 to 4

Name: _____

Date: _____

$$g = 10 \text{ m/s}^2$$

Part A. Multiple Choice

- Which pair of quantities consists of two scalars?
 - velocity and speed
 - displacement and distance
 - speed and mass
 - acceleration and time
- A boat is steered due north and moves at 4.0 m/s relative to the water. The water flows due east at 3.0 m/s relative to the bank. Relative to the bank, the speed of the boat is
 - 1.0 m/s
 - 5.0 m/s
 - 7.0 m/s
 - 3.5 m/s
- A block has been given a push and is now sliding to the right across a rough level floor, with nothing but the floor in contact with it. The forces exerted on the block are
 - gravity, the normal force and friction
 - gravity, the normal force, friction and a forward force
 - gravity and friction only
 - the normal force and friction only
- A 5.0 kg block is on a frictionless incline that makes 53° with the horizontal, with $\sin 53^\circ = 0.80$ and $\cos 53^\circ = 0.60$. The component of the gravitational force on the block along the incline is
 - 30 N
 - 40 N
 - 50 N
 - 25 N
- A ball thrown straight up is momentarily at rest at the highest point of its flight. At that instant the net force exerted on the ball
 - is zero, since the ball is at rest there
 - is zero, since the velocity has stopped changing
 - has magnitude mg and points downward
 - has magnitude mg and points upward
- An ideal spring of force constant 200 N/m hangs from a ceiling. A 2.0 kg block is attached to its lower end and hangs at rest. The spring has stretched by
 - 0.10 m
 - 0.020 m
 - 1.0 m
 - 10 m
- An object moves in a circle of radius r at constant speed. The speed is then doubled while the radius stays the same. The magnitude of its centripetal acceleration becomes
 - twice as large
 - four times as large
 - the same
 - half as large

8. The speed of a car is doubled and its mass does not change. Its translational kinetic energy becomes
- A. twice as large
 - B. four times as large
 - C. half as large
 - D. unchanged
9. Several forces act on a system as it moves. The total mechanical energy of the system does not change if
- A. the net external force on the system is zero at every instant
 - B. no force of any kind acts on any part of the system
 - C. every part of the system moves at a constant speed
 - D. only conservative forces do work on the system
10. A motor raises a 50 kg crate straight up at a steady 0.40 m/s. The power the motor delivers is
- A. 20 W
 - B. 125 W
 - C. 200 W
 - D. 500 W
11. Two vectors have equal magnitude V . Their vector sum also has magnitude V . The angle between the two vectors is
- A. 120°
 - B. 60°
 - C. 90°
 - D. 180°
12. A particle on the x axis has acceleration $a(t) = (6 - 4t) \text{ m/s}^2$, with t in seconds, and velocity 2.0 m/s at $t = 0$. Its velocity at $t = 3.0 \text{ s}$ is
- A. zero
 - B. -6.0 m/s
 - C. 20 m/s
 - D. 2.0 m/s
13. A graph of position against time is concave up everywhere, and its slope is negative everywhere shown. During that time the particle is
- A. moving in the negative direction and slowing down
 - B. moving in the negative direction and speeding up
 - C. moving in the positive direction and slowing down
 - D. moving in the positive direction and speeding up
14. Two projectiles are launched from level ground at the same speed, one at 30° and one at 60° above the horizontal. Compared with the 30° launch, the 60° launch has
- A. the same range and a greater maximum height
 - B. a greater range and the same maximum height
 - C. the same range and the same maximum height
 - D. a smaller range and a smaller maximum height
15. A shell launched from level ground explodes in midair into many fragments. Air resistance is negligible. Until the first fragment lands, the center of mass of the fragments
- A. stops at the point where the shell exploded
 - B. continues along the path the shell was following
 - C. falls straight down from the point of the explosion
 - D. moves off in the direction of the largest fragment

16. Two objects move along the x axis: a 1.0 kg object with $x_1 = 4t^2$ meters and a 3.0 kg object with $x_2 = 2t$ meters, with t in seconds. The speed of the center of mass of the pair at $t = 1.0$ s is
- A. 2.5 m/s
 - B. 5.0 m/s
 - C. 10 m/s
 - D. 3.5 m/s
17. A block slides down a frictionless incline. A student uses axes with x horizontal and y vertical instead of along and perpendicular to the surface. Which statement about that choice is correct?
- A. The acceleration comes out larger, since the slope is no longer an axis.
 - B. The acceleration comes out the same, and the normal force needs no resolving.
 - C. The axes cannot be used at all, since the normal force lies along neither one.
 - D. The acceleration comes out the same, and both of its components are nonzero.
18. A truck pushes a stalled car along a level road and both speed up. The force the truck exerts on the car and the force the car exerts on the truck are equal in magnitude. Both can still speed up because
- A. those two forces act on different objects and never cancel
 - B. the forward force on the car exceeds the backward force on the truck
 - C. the forward force on the car is exerted over a longer time interval
 - D. the pair cancels only for two objects of equal mass
19. A crate is pulled across a level floor at constant velocity by a horizontal rope. The rope is then released. Immediately afterward the crate
- A. keeps moving at constant velocity, since no new force appeared
 - B. slows down, since friction is the only horizontal force left
 - C. stops at the instant the rope is released
 - D. speeds up, since the rope is no longer holding it back
20. A 120 N sign hangs at rest from two cables, one on each side, each making 37° with the horizontal, with $\sin 37^\circ = 0.60$. The tension in each cable is
- A. 100 N
 - B. 60 N
 - C. 120 N
 - D. 150 N
21. A 5.0 kg object moves in the xy plane with $\vec{r}(t) = (t^3 \hat{i} + 4t^2 \hat{j})$ meters, with t in seconds. The magnitude of the net force on it at $t = 1.0$ s is
- A. 10 N
 - B. 30 N
 - C. 70 N
 - D. 50 N
22. An astronaut inside a space station in a low circular orbit floats freely rather than resting on the floor. The best account of this is that
- A. the gravitational force on the astronaut is zero at that altitude
 - B. the orbital speed cancels the gravitational force on the astronaut
 - C. an outward force balances the gravitational force on the astronaut
 - D. the astronaut and the station have the same acceleration

23. A crate on a level floor is pushed with a horizontal force raised slowly from zero until the crate begins to slide. For this crate $\mu_k < \mu_s$. Just after sliding begins, the friction force on the crate is
- A. larger than it was an instant earlier
 - B. smaller than it was an instant earlier
 - C. the same as it was an instant earlier
 - D. zero, because the crate is now moving
24. An ideal spring stores 2.0 J when it is stretched 0.10 m from its natural length. The energy it stores when it is stretched 0.30 m from that length is
- A. 6.0 J
 - B. 54 J
 - C. 18 J
 - D. 9.0 J
25. The position of a 2.0 kg particle is $x = t^2 + 2t$, in meters with t in seconds. Its translational kinetic energy at $t = 2.0$ s is
- A. 16 J
 - B. 18 J
 - C. 36 J
 - D. 72 J
26. A force $F(x) = \frac{4}{x^2}$ N acts along the x axis. The work it does on a particle that moves from $x = 1.0$ m to $x = 2.0$ m is
- A. 0.50 J
 - B. 1.0 J
 - C. 2.0 J
 - D. 4.0 J
27. A particle moves along x with $U(x) = 2x^3 - 9x^2 + 12x$ J, with x in meters. Which value of x is an unstable equilibrium point?
- A. $x = 0$
 - B. $x = 1.0$ m
 - C. $x = 2.0$ m
 - D. $x = 3.0$ m
28. A 2.0 kg cart starts from rest at $t = 0$ and its acceleration is $a(t) = 3t$ m/s², with t in seconds. Its momentum at $t = 2.0$ s is
- A. 12 kg·m/s
 - B. 6.0 kg·m/s
 - C. 18 kg·m/s
 - D. 24 kg·m/s
29. Two carts of unequal mass collide on a level frictionless track. The velocity of the center of mass of the two cart system
- A. is zero throughout
 - B. changes while the carts are touching and then returns to its old value
 - C. is the same before, during and after the collision
 - D. reverses when the carts bounce apart
30. Two carts of equal mass m approach each other on a level track, each with speed v . Which outcome is consistent with an elastic collision?
- A. both carts come to rest
 - B. they stick together and move off at speed v
 - C. one cart stops and the other moves off at speed $2v$
 - D. each cart rebounds with its original speed

31. For a particle moving along a line, the average velocity over an interval equals the instantaneous velocity at the midpoint of that interval for every motion in which
- A. the speed increases throughout the interval
 - B. the acceleration is constant
 - C. the particle starts from rest
 - D. the velocity changes sign exactly once
32. A ball is launched from level ground at 25 m/s at 53° above the horizontal, with $\sin 53^\circ = 0.80$. The ball is 20 m above the ground
- A. at $t = 1.0$ s and at $t = 3.0$ s
 - B. at $t = 2.0$ s and at $t = 4.0$ s
 - C. only at $t = 2.0$ s
 - D. at no time during the flight
33. A book rests on a level table. Under Newton's third law, the force that pairs with the gravitational force the Earth exerts on the book is
- A. the normal force the table exerts on the book
 - B. the normal force the book exerts on the table
 - C. the gravitational force the book exerts on the Earth
 - D. the gravitational force the Earth exerts on the table
34. A planet has twice the radius of the Earth and the same average density. The gravitational field strength at its surface, compared with the value at the surface of the Earth, is
- A. twice as large
 - B. four times as large
 - C. the same
 - D. half as large
35. A 2.0 kg block sits on a 4.0 kg block that rests on a frictionless floor, with $\mu_s = 0.50$ between the blocks. The largest horizontal force on the lower block that keeps the two moving together is
- A. 10 N
 - B. 60 N
 - C. 20 N
 - D. 30 N
36. A 5.0 kg block is pushed 4.0 m up a 37° incline at a steady speed by a force directed up the incline. The coefficient of kinetic friction is 0.25. Take $\sin 37^\circ = 0.60$ and $\cos 37^\circ = 0.80$. That force does
- A. 160 J
 - B. 120 J
 - C. 140 J
 - D. 200 J
37. A ball is thrown from the ground at 20 m/s at 37° above the horizontal. Air resistance is negligible. Its speed when it is 5.0 m above the ground is
- A. 10 m/s
 - B. 15 m/s
 - C. 17 m/s
 - D. 19 m/s

38. A ball of mass m is tossed straight up with speed v_0 and caught at the same height. Neglecting air resistance, the impulse delivered to the ball by gravity over the whole flight
- A. is zero, because the ball returns with the speed it left with
 - B. points downward and has magnitude $2mv_0$
 - C. points upward on the way up and downward on the way down
 - D. is zero, because the ball returns to the point it started from
39. A 1.0 kg block moving at 6.0 m/s strikes a stationary 2.0 kg block and the two stick together. They then slide on a level surface whose coefficient of kinetic friction is 0.25. They slide
- A. 0.80 m
 - B. 1.6 m
 - C. 2.4 m
 - D. 7.2 m
40. A light ball collides head on and elastically with a stationary ball of much greater mass. Immediately afterward,
- A. the light ball nearly stops, and the heavy ball moves off slowly in the same direction
 - B. the light ball rebounds with nearly its original speed, and the heavy ball barely moves
 - C. both balls move forward together at roughly half the original speed of the light ball
 - D. the light ball keeps going forward at nearly its original speed, and so does the heavy ball
41. A particle on the x axis has $a(t) = -6t \text{ m/s}^2$, with t in seconds. At $t = 0$ it is at the origin moving at 12 m/s. Its greatest positive displacement from the origin is
- A. 24 m
 - B. 8.0 m
 - C. 16 m
 - D. 12 m
42. Two balls are thrown from the same point at the same instant with different initial velocities. Air resistance is negligible. While both are in flight, the velocity of one ball relative to the other
- A. grows in magnitude as they fall
 - B. keeps the same magnitude but turns steadily
 - C. points vertically downward
 - D. stays constant in magnitude and direction
43. Blocks of 3.0 kg and 1.0 kg hang from the two ends of a light string that passes over a light frictionless pulley. The pulley hangs from a spring scale. The scale reads
- A. 40 N
 - B. 20 N
 - C. 30 N
 - D. 15 N
44. An object moving through a fluid obeys $m \, dv/dt = mg - bv$, taking down as positive. It is thrown straight down at twice its terminal speed. Immediately after release its acceleration is
- A. zero
 - B. g , directed downward
 - C. $2g$, directed upward
 - D. g , directed upward

45. A projectile is launched from level ground at 20 m/s at 53° above the horizontal, with $\cos 53^\circ = 0.60$. At the highest point of its path, the radius of curvature of the path is
- A. 14.4 m
 - B. 40 m
 - C. 25.6 m
 - D. 20 m
46. A 2.0 kg particle moves along x with $U(x) = (x^2 - 4)^2$ J, with x in meters. It is released from rest at $x = 1.0$ m. Its greatest speed in the motion that follows is
- A. 2.1 m/s
 - B. 4.0 m/s
 - C. 6.0 m/s
 - D. 3.0 m/s
47. A block slides down a rough incline at a steady speed. Which statement about the power delivered to the block is correct?
- A. gravity delivers positive power and friction delivers negative power of equal magnitude
 - B. gravity and friction both deliver negative power, so the block must be slowing down
 - C. gravity delivers no power, because the kinetic energy of the block is not changing
 - D. the normal force and friction deliver powers that are equal and opposite in sign
48. A small canister at rest at the edge of a cliff bursts into two unequal pieces that fly off horizontally in opposite directions. Neglecting air resistance,
- A. they land at the same time and at the same distance from the base of the cliff
 - B. the lighter piece lands first, because it leaves the cliff with the greater speed
 - C. they land at the same time, and the lighter piece lands farther from the base
 - D. the heavier piece lands farther from the base, because it carries more momentum
49. A chain of mass 2.0 kg per meter lies coiled on the floor. One end is lifted straight up at a steady 3.0 m/s. When 1.5 m of chain is off the floor, the upward force applied to the end is
- A. 30 N
 - B. 39 N
 - C. 66 N
 - D. 48 N
50. A spring with $F(x) = -400x$ N is compressed 0.20 m behind a 1.0 kg cart on a level frictionless track. Released, the cart then strikes a stationary 3.0 kg cart and the two stick together. The kinetic energy of the pair is
- A. 1.0 J
 - B. 8.0 J
 - C. 2.0 J
 - D. 4.0 J

Part B. Reasoning

1. Two vectors have magnitudes 3.0 and 4.0. Explain why those two numbers do not usually give us the magnitude of the resultant vector, and give the largest and the smallest values it can take.
2. An elevator rises at a steady 2.0 m/s on a single cable, and a student says the tension must exceed the weight because the elevator is moving upward. Correct the student.
3. An ideal spring obeys $\vec{F}_s = -k\Delta\vec{x}$. Explain what the minus sign represents, and state the force when a spring of $k = 200$ N/m is stretched 0.050 m.
4. A student says that a 2.0 kg object moving at 3.0 m/s in the $-x$ direction has a kinetic energy of -9.0 J. Correct the student.
5. A particle moves along the x -axis with position $x(t)$. Explain why knowing that the velocity is zero at one instant tells you nothing about the acceleration at that same instant.
6. The velocity of a particle runs along a straight line on a velocity against time graph from $+6.0$ m/s at $t = 0$ to -6.0 m/s at $t = 3.0$ s. State what the slope gives, and explain why the area between the graph and the time axis gives the displacement rather than the distance traveled.
7. A 3.0 kg book lies on top of a 6.0 kg box, and the box sits on a level floor. Name every force exerted on the book along with which object exerts it, and its magnitude.
8. A 1.0 kg book rests on a table. Explain why the upward force the table exerts on the book and the downward force the book exerts on the table cannot be treated as a pair that cancels in a Newton's second law equation.
9. A student solves every dynamics problem by finding the acceleration and then using the constant acceleration equations. Explain why that method fails for a force that depends on the velocity, and what has to be done instead.
10. A 1200 kg car rounds a level curve of radius 50 m at 10 m/s, and a student adds a centripetal force of 2400 N to the free body diagram of the car. Correct the student.
11. A car braking with a constant force stops from 20 m/s in 25 m. Explain why that same braking force needs 100 m to stop it from 40 m/s.
12. State the condition under which the mechanical energy of a chosen system stays constant, and explain why the answer depends on where the boundary of the system is drawn.
13. An object released from rest in a fluid obeys $m dv/dt = mg - bv$. Explain why its speed approaches the terminal value without ever reaching it at any finite time, and use calculus to prove your point.
14. A particle moves in the potential energy $U(x) = 5x^2 - 20x$ J, with x in meters. Locate the equilibrium point, and explain what the second derivative of U there tells you about the motion of the particle near it.
15. Starting from the definition of the center of mass, show that the total momentum of a system is $M\vec{v}_{cm}$, and explain why $\sum \vec{F}_{ext} = M\vec{a}_{cm}$ comes after it.
16. A 2.0 kg cart moving at 5.0 m/s collides elastically with a 3.0 kg cart at rest on a level track. Show that the speed at which the two carts separate equals the speed at which they approached, and explain where that result comes from.
17. A particle moves with constant speed along a curved path. Explain why its acceleration has to be perpendicular to its velocity at every instant, and state the size of that acceleration on a circle of radius 2.0 m at 4.0 m/s.
18. A satellite is moved from a circular orbit of radius r , where its speed is 8.0 km/s, to a circular orbit of radius $4r$. A student says it must move faster there because it has been given more energy, so explain what is right and what is wrong in that claim.
19. A 1000 kg car is modeled as receiving a constant 30 kW from its engine on a level road with no resistance, and a student asks for its acceleration at the instant it starts from rest. Explain what is wrong with the question.
20. A 2.0 kg cart moving east at 3.0 m/s meets a 2.0 kg cart at rest on a level frictionless track and the two stick together. Compare what a ground observer and an observer moving east at 3.0 m/s find for the total momentum before and after, and explain why each of them is entitled to say it is conserved.

Part C. Problems

- A surveyor walks 90 m due east along one edge of a field and then 120 m due north along the next edge.
 - Derive an expression for the magnitude of the total displacement in terms of the two legs, then calculate it.
 - Calculate the angle of the total displacement, measured north of east.
 - Calculate the total distance the surveyor walked, and state why it is not the same as the answer to part (a).
- A 5.0 kg crate is pulled across a level floor by a horizontal force of 20 N. The coefficient of kinetic friction between the crate and the floor is 0.25. Take $g = 10 \text{ m/s}^2$.
 - Derive an expression for the magnitude of the kinetic friction force on the crate in terms of μ_k , m and g , then calculate it.
 - Calculate the magnitude of the acceleration of the crate.
 - The crate starts from rest. Calculate its speed after it has moved 3.0 m.
- A 5.0 kg block slides along a level frictionless floor with kinetic energy 90 J.
 - Derive an expression for the speed of the block in terms of its kinetic energy and its mass, then calculate it.
 - Calculate the magnitude of the momentum of the block.
 - The block is later moving at half that speed. Calculate its kinetic energy then.
- A 3.0 kg puck slides across a level frictionless table with velocity $\vec{v} = (4.0\hat{i} + 3.0\hat{j}) \text{ m/s}$.
 - Calculate the x component of the momentum of the puck.
 - Calculate the magnitude of the momentum of the puck.
 - Calculate the kinetic energy of the puck.
- A particle moves along the x -axis with $x(t) = (2t^3 - 9t^2 + 12t) \text{ m}$, with t in seconds and $t \geq 0$.
 - Derive an expression for the velocity as a function of time, then calculate the earliest time at which the particle is momentarily at rest.
 - Derive an expression for the acceleration as a function of time, then calculate it at $t = 3.0 \text{ s}$.
 - Calculate the displacement of the particle between $t = 1.0 \text{ s}$ and $t = 2.0 \text{ s}$, and explain what its sign tells you.
- A 4.0 kg object rests at the origin on a level frictionless surface. From $t = 0$ onward a net force $F(t) = (24 - 6t) \text{ N}$ acts on it along the x -axis, with t in seconds. The object starts from rest.
 - Derive an expression for the acceleration as a function of time, then calculate the time at which the acceleration is zero.
 - Derive an expression for the velocity as a function of time, then calculate the velocity at $t = 4.0 \text{ s}$.
 - Calculate the time at which the object is momentarily at rest again.
- A 2.0 kg ball falls through air that exerts on it a resistive force of magnitude bv^2 , directed opposite to the velocity, with $b = 0.80 \text{ kg/m}$. Take $g = 10 \text{ m/s}^2$ and take downward as positive.
 - Write the differential equation that governs the speed of the ball, then derive an expression for its terminal speed in terms of m , g and b and calculate it.
 - Calculate the magnitude of the acceleration of the ball at the instant its speed is 3.0 m/s.
 - Derive an expression for the speed at which the acceleration has fallen to half of g , in terms of v_T , then calculate it.
- A 0.50 kg ball on the end of a light string of length 0.80 m is swung in a vertical circle. Take $g = 10 \text{ m/s}^2$.
 - Derive an expression for the tension in the string at the lowest point of the circle in terms of m , v , r and g , then calculate it for a speed of 4.0 m/s there.
 - Derive an expression for the smallest speed at the highest point for which the string stays taut, in terms of g and r , then calculate it.
 - Calculate the tension at the highest point when the ball passes it at 6.0 m/s.
- A 4.0 kg block starts from rest at $x = 0$ on a level frictionless surface. A single horizontal force $F(x) = 12 - 3x^2 \text{ N}$ acts on it along the x axis, with x in meters.
 - Derive an expression for the work done by the force from $x = 0$ to $x = X$, then calculate it for $X = 2.0 \text{ m}$.
 - Calculate the speed of the block at $x = 2.0 \text{ m}$.
 - Calculate the position beyond $x = 2.0 \text{ m}$ at which the block is again momentarily at rest.

- 10.** A 0.25 kg particle moves along the x axis in the potential energy $U(x) = 2x^3 - 24x$ J, with x in meters. The total energy of the particle is -20 J.
- Derive an expression for the force on the particle, then calculate it at $x = 1.0$ m.
 - Calculate the position of the equilibrium point that lies at positive x .
 - Calculate the speed of the particle at $x = 1.0$ m.
- 11.** A 2.5 kg object slides along a level frictionless surface in the $+x$ direction at 4.0 m/s. Starting at $t = 0$ a net force $F(t) = -6t$ N acts on it along the x axis, with t in seconds.
- Derive an expression for the impulse delivered between $t = 0$ and $t = T$, then calculate it for $T = 2.0$ s.
 - Calculate the velocity of the object at $t = 2.0$ s.
 - Calculate the time at which the object is momentarily at rest.
- 12.** A 20 kg cart coasts along level frictionless track at 6.0 m/s. Starting at $t = 0$ sand pours vertically into the cart at a steady 4.0 kg/s. The sand has no horizontal velocity before it lands.
- Derive an expression for the speed of the loaded cart as a function of time, then calculate it at $t = 5.0$ s.
 - Calculate the kinetic energy lost between $t = 0$ and $t = 5.0$ s.
 - Derive an expression for the acceleration of the loaded cart as a function of time, then calculate it at $t = 5.0$ s.
- 13.** A river of width 120 m flows due east at 2.0 m/s. A small boat travels at 2.5 m/s relative to the water.
- The driver points the boat due north, straight across the current. Derive an expression for the time to cross in terms of the width and the speed of the boat relative to the water, then calculate it.
 - Calculate how far downstream of the starting point the boat reaches the far bank.
 - The driver now aims the boat upstream at the angle that makes it travel straight across. Derive an expression for the time to cross in terms of the width and the two speeds, then calculate it.
- 14.** A thin rod lies along the x -axis from $x = 0$ to $x = L$. Its linear mass density is $\lambda(x) = \lambda_0 \left(1 + \frac{x^2}{L^2}\right)$, with $\lambda_0 = 0.75$ kg/m and $L = 1.6$ m.
- Derive an expression for the total mass of the rod in terms of λ_0 and L , then calculate it.
 - Derive an expression for the position of the center of mass in terms of L , then calculate it.
 - Calculate the mass of the half of the rod nearer the origin, and explain how it confirms the answer to part (b).
- 15.** A 3.0 kg object moves along the x axis with position $x(t) = 2t^3 - 3t^2$ meters, with t in seconds.
- Derive an expression for the net force on the object as a function of time, then calculate it at $t = 2.0$ s.
 - Derive an expression for the instantaneous power delivered by the net force, then calculate it at $t = 2.0$ s.
 - Calculate the magnitude of the impulse delivered to the object between $t = 0$ and $t = 2.0$ s.
- 16.** On a level track a 0.80 kg block moving at 10 m/s collides with a 1.2 kg block at rest, and the two move off together. The track is frictionless where the collision happens, but the pair then slides onto a rough stretch where the coefficient of kinetic friction is 0.25. Take $g = 10$ m/s².
- Derive an expression for the speed of the pair just after the collision in terms of the two masses and the initial speed, then calculate it.
 - Derive an expression for the distance the pair slides on the rough stretch before stopping, in terms of v , μ and g , then calculate it.
 - Calculate the kinetic energy lost in the collision itself.
- 17.** A particle leaves the origin at $t = 0$ with velocity $\vec{v}(0) = 4.0\hat{i}$ m/s. From then on its acceleration is $\vec{a}(t) = 6t\hat{j}$ m/s², with t in seconds. The motion is in the xy -plane.
- Derive expressions for the two velocity components as functions of time, then calculate the speed of the particle at $t = 2.0$ s.
 - Derive expressions for the two position coordinates as functions of time, then calculate the distance of the particle from the origin at $t = 2.0$ s.
 - Calculate the time at which the velocity of the particle makes an angle of 45° with the x -axis.

18. A 1.2 kg puck slides along a level frictionless track through a fluid that exerts on it a resistive force $\vec{F}_r = -b\vec{v}$, with $b = 0.30$ kg/s. Gravity and the normal force balance, so the resistive force is the only horizontal force. The puck passes the origin at $t = 0$ moving in the $+x$ direction at 6.0 m/s.

- (a) Write the differential equation for the speed, solve it, and derive an expression for $v(t)$ in terms of m , b and the initial speed, then calculate the speed at $t = 4.0$ s.
- (b) Derive an expression for the total distance the puck travels, in terms of m , b and the initial speed, then calculate it.
- (c) Calculate the time at which the puck has covered half of that total distance.

19. A 1.0 kg cart on a level frictionless track is held against a spring that is not linear: when the spring is compressed a distance x it pushes outward with a force of magnitude $600x^2$ N, with x in meters. The cart is released from a compression of 0.30 m, leaves the spring, and then collides with a 3.0 kg cart at rest. The two carts move off together.

- (a) Derive an expression for the energy stored in the spring at a compression x_0 , then calculate it for $x_0 = 0.30$ m.
- (b) Calculate the speed of the two carts after the collision.
- (c) Calculate the kinetic energy lost in the collision.

20. A 2.0 kg puck slides in the $+x$ direction at 8.0 m/s along a level frictionless surface. Between $x = 0$ and $x = 2.0$ m a single horizontal force acts on it, $F(x) = -3x^2$ N, with x in meters. Beyond $x = 2.0$ m no horizontal force acts.

- (a) Derive an expression for the work done by that force from $x = 0$ to $x = X$, then calculate it for the whole region, $X = 2.0$ m.
- (b) Calculate the speed of the puck at $x = 2.0$ m.
- (c) Calculate the magnitude of the impulse this force delivered to the puck.

AP Physics C: Mechanics, Equation Sheet

The constants, conventions and equations on this sheet are those the College Board provides on the AP Physics exam reference information.

CONSTANTS AND CONVERSION FACTORS

$$G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

universal gravitational constant

$$g = 9.8 \text{ m/s}^2$$

magnitude of the acceleration due to gravity at the surface of the Earth

$$g = 9.8 \text{ N/kg}$$

magnitude of the gravitational field strength at the surface of the Earth

CONVENTIONS

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT SYMBOLS

hertz	Hz
newton	N
joule	J
second	s
kilogram	kg
watt	W
meter	m

TRIGONOMETRIC FUNCTIONS AT COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

$$s = r\theta$$

Rectangular solid

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = a/c$$

$$\cos \theta = b/c$$

$$\tan \theta = a/b$$

KINEMATICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\Delta x = \int v_x(t) dt$$

$$\Delta v_x = \int a_x(t) dt$$

CENTER OF MASS

$$\vec{x}_{\text{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\vec{r}_{\text{cm}} = \frac{\int \vec{r} dm}{\int dm}$$

$$\lambda = \frac{d}{d\ell} m(\ell)$$

$$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

FORCE AND TRANSLATIONAL DYNAMICS

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$|\vec{F}_g| = G \frac{m_1 m_2}{r^2}$$

$$|\vec{F}_f| \leq |\mu \vec{F}_N|$$

$$\vec{F}_s = -k\Delta\vec{x}$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$T = \frac{1}{f}$$

WORK, ENERGY AND POWER

$$K = \frac{1}{2}mv^2$$

$$W = \int_a^b \vec{F} \cdot d\vec{r}$$

$$\Delta K = \sum W_i = \sum F_{\parallel, i} d_i$$

$$\Delta U = - \int_a^b \vec{F}_{\text{cf}}(r) \cdot d\vec{r}$$

AP PHYSICS C: MECHANICS

$$F_x = -\frac{dU(x)}{dx}$$

$$U_s = \frac{1}{2}k(\Delta x)^2$$

$$U_G = -G\frac{m_1 m_2}{r}$$

$$\Delta U_g = mg\Delta y$$

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = \frac{dW}{dt}$$

LINEAR MOMENTUM

$$\vec{p} = m\vec{v}$$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t) dt = \Delta \vec{p}$$

TORQUE AND ROTATIONAL DYNAMICS

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$v = r\omega$$

$$a_T = r\alpha$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$I_{\text{tot}} = \sum I_i = \sum m_i r_i^2$$

$$I = \int r^2 dm$$

$$I' = I_{\text{cm}} + Md^2$$

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

ENERGY AND MOMENTUM OF ROTATING SYSTEMS

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$W = \int \tau \cdot d\theta$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$\Delta L = \int \tau dt$$

$$\Delta x_{\text{cm}} = r\Delta\theta$$

OSCILLATIONS

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_{\text{phys}} = 2\pi\sqrt{\frac{I}{mgd}}$$

$$x = x_{\text{max}} \cos(\omega t + \phi)$$

VECTORS

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

$$\vec{r} = (A\hat{i} + B\hat{j} + C\hat{k})$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax)$$

IDENTITIES

$$\log(a \cdot b^x) = \log a + x \log b$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

WHAT THE LETTERS MEAN

a	acceleration
d	distance
E	energy
f	frequency
F	force
I	rotational inertia
J	impulse
k	spring constant
K	kinetic energy
ℓ	length
L	angular momentum
m	mass
M	mass
p	momentum
P	power
r	radius, distance, or position
t	time
T	period
U	potential energy
v	velocity or speed
W	work
x	position or distance
y	vertical position
α	angular acceleration
θ	angular position
λ	linear mass density
μ	coefficient of friction
τ	torque
ϕ	phase angle
ω	angular frequency or angular speed