

Torque and Rotational Dynamics Practice Quiz

Name: _____

Date: _____

$$g = 10 \text{ m/s}^2$$

Part A. Multiple Choice

1. A grindstone turns about its spindle so that its angle, in radians, is $\theta(t) = (0.80 \text{ rad/s}^3)t^3$. The angular velocity of the stone at $t = 2.0 \text{ s}$ is
 - A. 2.4 rad/s
 - B. 9.6 rad/s
 - C. 6.4 rad/s
 - D. 3.2 rad/s
2. The angular velocity of a centrifuge rotor is $\omega(t) = (6.0 \text{ rad/s}^2)t - (1.5 \text{ rad/s}^3)t^2$. Between $t = 0$ and $t = 2.0 \text{ s}$ the rotor turns through an angle of
 - A. 6.0 rad
 - B. 12 rad
 - C. 16 rad
 - D. 8.0 rad
3. For a turning wheel, the area between the graph of angular acceleration against time and the time axis, taken over an interval, gives
 - A. the change in the angular velocity over that interval
 - B. the change in the angle over that interval
 - C. the average angular acceleration over that interval
 - D. the angular velocity at the end of that interval
4. A rotor starts from rest at $t = 0$ with $\alpha(t) = (3.0 \text{ rad/s}^3)t - 6.0 \text{ rad/s}^2$. The next instant at which it is momentarily at rest is
 - A. 6.0 s
 - B. it does not come to rest again
 - C. 4.0 s
 - D. 2.0 s
5. A disk has a negative angular velocity at every instant of an interval and a positive angular acceleration throughout. Over that interval the disk
 - A. comes to rest part way through and stays at rest
 - B. turns in one sense the whole time, and the magnitude of its angular velocity falls off
 - C. turns in one sense the whole time, and the magnitude of its angular velocity grows steadily
 - D. reverses its sense of turning part way through the interval
6. A conveyor roller of radius 0.12 m turns through 20 rad without slipping against the belt. The length of belt that passes over it is
 - A. 167 m
 - B. 0.0060 m
 - C. 20 m
 - D. 2.4 m
7. A wheel rolls along level ground without slipping. At one instant, the velocity of the point of the wheel that touches the ground is
 - A. twice the velocity of the center of the wheel

- B. directed straight down, into the ground
 - C. zero
 - D. equal to the velocity of the center of the wheel
8. A disk turns at a steady angular velocity. For a point on the disk away from the axis, the acceleration is
- A. not zero, and it points toward the axis
 - B. zero, because the angular velocity does not change
 - C. not zero, and it points along the direction of travel
 - D. zero everywhere except at the rim of the disk
9. A hoist drum of radius 0.15 m turns with $\theta(t) = (2.0 \text{ rad/s}^2)t^2$ and winds in a cable from its rim. The cable speed at $t = 3.0 \text{ s}$ is
- A. 12 m/s
 - B. 1.8 m/s
 - C. 0.60 m/s
 - D. 2.7 m/s
10. A cable unwinds from a drum that turns at a steady angular velocity. As cable comes off, the radius of the coil that remains shrinks. While that happens, the cable speed
- A. stays the same, because the angular velocity of the drum is held steady throughout
 - B. grows, because there is less cable left on the drum to drag along
 - C. stays the same, because the cable is not stretching anywhere
 - D. falls off, because the radius at which the cable leaves the coil is shrinking
11. A wrench is pulled so that it would turn a nut clockwise. Counterclockwise is taken as positive. The torque of that pull about the bolt is
- A. negative, because the sense of the turn it would produce is the negative sense
 - B. positive, because the magnitude of a torque is never a negative number
 - C. positive, because the pull is applied out along the handle of the wrench
 - D. negative only if the pull is perpendicular to the handle of the wrench
12. A light beam is hinged at its midpoint. A 20 N load hangs 0.30 m to the right of the hinge and a 12 N load hangs 0.60 m to the left. With counterclockwise positive, the net torque is
- A. 13.2 N·m
 - B. +7.2 N·m
 - C. +1.2 N·m
 - D. -1.2 N·m
13. A 35 N force is applied to a door at a point 0.90 m from the hinges, in a direction 120 degrees from the plane of the door. The magnitude of the torque about the hinges is
- A. 13.6 N·m
 - B. 27.3 N·m
 - C. 15.8 N·m
 - D. 31.5 N·m
14. A force acts on a rigid body at one point. The point of application is then moved to another point on the same line of action, with the force unchanged. The torque about a fixed axis is
- A. greater, because the new point of application sits farther from the chosen axis than before
 - B. zero, because the force now passes through a different point of the body
 - C. the same only when the chosen axis happens to lie on that line
 - D. the same, because the perpendicular distance from the axis to that line is unchanged

15. Two forces of equal magnitude act on a plate in opposite directions, along two parallel lines a fixed distance apart. The net torque about an axis perpendicular to the plate is
- A. different for every choice of axis, since each torque depends on the axis
 - B. zero about the center of mass of the plate, and not zero about any other axis whatsoever
 - C. the same about every such axis, and set by the force and the distance between the lines
 - D. zero, because the two forces are equal in magnitude and opposite in direction
16. A thin uniform bar of mass 2.4 kg and length 1.5 m swings about one end, with the axis at right angles to the bar. Evaluating $I = \int r^2 dm$ for the bar gives
- A. $1.8 \text{ kg}\cdot\text{m}^2$
 - B. $0.45 \text{ kg}\cdot\text{m}^2$
 - C. $5.4 \text{ kg}\cdot\text{m}^2$
 - D. $0.90 \text{ kg}\cdot\text{m}^2$
17. A thin rod has a linear density that grows steadily from one end to the other. Set against the rotational inertia about an axis at the heavy end, the value about a parallel axis at the light end is
- A. the same, because the two axes are parallel to each other
 - B. greater, because more of the mass then sits far from the axis
 - C. smaller, because the light end carries very little mass near the axis
 - D. the same, because the rod and its total mass are the same either way
18. A uniform hatch cover of mass 4.0 kg has $I_{\text{cm}} = 0.50 \text{ kg}\cdot\text{m}^2$ about an axis through its center of mass, at right angles to the cover. About a parallel axis 0.30 m from that one, its rotational inertia is
- A. $0.36 \text{ kg}\cdot\text{m}^2$
 - B. $1.70 \text{ kg}\cdot\text{m}^2$
 - C. $0.86 \text{ kg}\cdot\text{m}^2$
 - D. $0.50 \text{ kg}\cdot\text{m}^2$
19. A tapered baton of mass 3.0 kg and length $L = 2.0 \text{ m}$ has $\lambda(x) = \lambda_0(1 - x/L)$, with x measured from the thick end. Its rotational inertia about an axis at the thick end, at right angles to it, is
- A. $4.0 \text{ kg}\cdot\text{m}^2$
 - B. $1.0 \text{ kg}\cdot\text{m}^2$
 - C. $12 \text{ kg}\cdot\text{m}^2$
 - D. $2.0 \text{ kg}\cdot\text{m}^2$
20. A rigid body has the same rotational inertia about two different parallel axes, neither of which passes through its center of mass. That happens when
- A. the two axes sit the same distance from the center of mass
 - B. the two axes sit as far apart as each one sits from the center of mass
 - C. the body is uniform and its mass is spread evenly
 - D. never, since each axis gives a value of its own
21. A rigid body is in rotational equilibrium. That statement tells you
- A. the forces exerted on it add to zero
 - B. the torques exerted on it add to zero
 - C. it is not turning at all at that moment
 - D. it is at rest and will stay at rest
22. A light wheelbarrow carries a 40 kg load whose weight acts 0.50 m behind the axle. The handles are 1.20 m behind the axle. Taking $g = 10 \text{ m/s}^2$, the upward force needed at the handles is
- A. 960 N

- B. 200 N
C. 167 N
D. 400 N
23. In a problem about a body in equilibrium, torques may be taken about any axis. That freedom rests on the fact that
A. the torque of any single force does not depend on the axis that it is taken about
B. only an axis through the center of mass gives a sum of zero
C. every force in such a problem acts along a line through one point
D. the net torque about every axis is zero when both equilibrium conditions hold
24. A uniform sign of mass 6.0 kg and width 1.40 m is suspended by two short wires, one 0.20 m in from its left edge and the other 0.30 m in from its right edge. With $g = 10 \text{ m/s}^2$, the left wire carries
A. 60 N
B. 26.7 N
C. 33.3 N
D. 30 N
25. The torques on a plate about an axis through one corner add to zero, and the net force on the plate is not zero. About an axis through a second corner, the sum of the torques is
A. not zero, unless the line joining the two corners runs along the net force
B. zero as well, because rotational equilibrium holds about every axis
C. zero as well, because the plate is rigid and holds its shape
D. not zero, whichever second corner is chosen for the axis
26. An attendant turns a heavy valve wheel. A steady net torque of 12 N·m gives the wheel an angular acceleration of 0.80 rad/s^2 . The rotational inertia of the wheel about the shaft it turns on is
A. $9.6 \text{ kg}\cdot\text{m}^2$
B. $0.067 \text{ kg}\cdot\text{m}^2$
C. $12 \text{ kg}\cdot\text{m}^2$
D. $15 \text{ kg}\cdot\text{m}^2$
27. A grindstone slows at a steady rate after its motor is switched off. From that observation alone it follows that
A. its rotational inertia is falling off as the grindstone slows
B. the net torque on it grows as the grindstone slows
C. a steady net torque acts on it, in the sense opposite to its turning
D. no net torque acts on it, since nothing is driving it any more
28. A rotor of rotational inertia $0.40 \text{ kg}\cdot\text{m}^2$ turns at 5.0 rad/s at $t = 0$ on a frictionless axle. A net torque $\tau(t) = (0.60 \text{ N}\cdot\text{m/s}^2)t^2$ then acts. Its angular velocity at $t = 2.0 \text{ s}$ is
A. 9.0 rad/s
B. 4.0 rad/s
C. 11 rad/s
D. 17 rad/s
29. A cord is wrapped around the rim of a uniform disk of mass 4.0 kg and radius 0.25 m that turns on a fixed axle, with $I = \frac{1}{2}MR^2$. The cord is pulled with a steady 6.0 N. The angular acceleration is
A. 1.5 rad/s^2
B. 12 rad/s^2
C. 6.0 rad/s^2
D. 24 rad/s^2

30. A uniform rod is hinged at one end and held horizontal, then released. Just after release, the angular acceleration of the rod about the hinge
- A. grows in proportion to the mass of the rod
 - B. does not depend on the length of the rod
 - C. is zero, because the hinge holds the rod up
 - D. does not depend on the mass of the rod
- 31.* A turbine shaft is at rest at $t = 0$ and has $\alpha(t) = (6.0 \text{ rad/s}^4)t^2 - (12 \text{ rad/s}^3)t$ after that. Its average angular velocity over the first 3.0 s is
- A. -13.5 rad/s
 - B. -6.75 rad/s
 - C. -4.5 rad/s
 - D. 0
- 32.* A wrench is pulled at a fixed point on its handle with a force of fixed magnitude. The pull is then aimed so that it makes a greater angle with the handle than before. The torque about the bolt
- A. may grow or may fall off, since the torque is greatest at a right angle to the handle
 - B. grows, since the part of the pull across the handle grows with the angle
 - C. falls off, since less of the pull then lies along the handle
 - D. does not change, since neither the magnitude of the pull nor the point it acts at has changed
- 33.* A uniform rod of mass 3.0 kg and length 1.2 m carries a 1.0 kg bead fixed at one end. The rod turns about an axis perpendicular to it, 0.30 m from the other end. Its rotational inertia about that axis is
- A. $0.63 \text{ kg}\cdot\text{m}^2$
 - B. $1.44 \text{ kg}\cdot\text{m}^2$
 - C. $2.88 \text{ kg}\cdot\text{m}^2$
 - D. $1.17 \text{ kg}\cdot\text{m}^2$
- 34.* A uniform trapdoor of mass 12 kg and width 1.0 m is hinged at one edge and held horizontal by a rope at the free edge, 30 degrees from the door. A 5.0 kg bucket hangs 0.25 m from the hinge. With $g = 10 \text{ m/s}^2$, the tension is
- A. 120 N
 - B. 83.7 N
 - C. 72.5 N
 - D. 145 N
- 35.* A flywheel, $I = 0.60 \text{ kg}\cdot\text{m}^2$, is held at rest by 1.8 N·m of friction as a torque $\tau = (0.90 \text{ N}\cdot\text{m/s})t$ builds from $t = 0$. The friction stays at 1.8 N·m once the wheel turns. At $t = 6.0 \text{ s}$, ω is
- A. 27 rad/s
 - B. 24 rad/s
 - C. 12 rad/s
 - D. 9.0 rad/s

* Challenging.

Part B. Reasoning

1. Explain why the angular velocity of a turning body at one instant is the derivative of the angle with respect to time, and not the angle divided by the time that has elapsed.
2. Explain what the integral of the angular acceleration over an interval gives for a turning rotor, and describe what the problem has to supply besides that integral before the angular velocity at the end of the interval is known.
3. A rotor speeds up with an angular acceleration that grows as time passes. Explain why the equations written for a steady angular acceleration do not apply to it, and describe what takes their place.
4. Describe what the angle, the angular velocity and the angular acceleration each report about a turning body, and explain why the value of one of the three at an instant does not give the other two.
5. Explain why each linear quantity of a point on a rotating rigid body carries exactly one factor of its distance from the axis, and describe how that shows in the tangential acceleration.
6. Compare the tangential acceleration and the radial acceleration of a point on a disk whose angular velocity is growing, and explain why only one of the two falls to zero once the angular velocity is steady.
7. Explain why the arc length, and not the straight line joining the starting and ending positions, gives the distance that a point on a turning disk actually travels.

8. Explain why a torque always has to be stated about a named axis, and describe what can happen to the torque of one given force when a different axis is named instead.
9. Describe what the sign of a torque reports about a force acting on a body that can turn, and explain why that sign has no meaning until a positive sense of turning has been stated.
10. Two pushes of the same magnitude act on a bar at the same distance from its pivot, yet their torques differ in magnitude. Explain how that is possible, and describe the case that gives the greatest torque.
11. Explain why the rotational inertia of a continuous body is written as the integral of $r^2 dm$, and state what the r in that integral measures.
12. Explain why the parallel axis theorem compares only axes that are parallel to each other, and describe what the distance that appears in the theorem measures.
13. Describe what the rotational inertia of a rigid body depends on, and explain why neither the angular velocity the body happens to have nor the torque applied to it appears anywhere in that list.
14. Describe how the integral for the rotational inertia of a thin rod changes when the rod is made so that its linear density varies along its length, and explain which factor in the integral carries that change.

15. Explain why a body may have no net force on it and still begin to turn, and describe what has to be true of the forces acting on it for that to happen.
16. Two movers carry a uniform steel pipe, one lifting at one end and the other lifting at a point in from the far end. Explain why the two do not lift equal shares of the weight.
17. A heavy shutter hangs from two hinges on one side of a window frame. Explain why the upper hinge pulls the shutter toward the frame while the lower hinge pushes it away.
18. Describe what a net torque that grows with time does to the angular velocity of a flywheel on a fixed axle, and explain why that angular velocity has to be found by integrating.
19. Compare Newton's second law in its rotational form with the same law in its linear form, and describe which quantity in the rotational form plays the part that mass plays in the linear form.
20. Explain why a net torque applied to a wheel that is already turning changes its angular velocity rather than setting it, and describe what the angular velocity does once the net torque falls to zero.
- 21.* A rotor turns with an angular acceleration that varies with time. A student states that its average angular velocity over an interval is half the sum of the angular velocities at the two ends. Explain the flaw.

- 22.* A thin uniform rod turns about an axis at its midpoint, at right angles to the rod, and its rotational inertia there is known. A student states that the value about an axis running along the rod is the same. Explain the flaw.
- 23.* For a flat equilibrium problem a student writes a torque equation about one axis, a second torque equation about a different axis, and the two force equations, and states that this gives four independent equations. Explain the flaw.
- 24.* A flywheel starts from rest and the net torque on it grows steadily from zero. A student multiplies the angular acceleration at the final instant by the elapsed time to get the angular velocity. Explain the flaw.
- 25.* A rigid bar is built from sections of different mass. Explain why the torque of its weight about any axis may be found by treating the whole weight as acting at one point, and state which point that is.

* Challenging.

Part C. Problems

1. A lathe spindle turns so that its angle, in radians, is $\theta(t) = (4.0 \text{ rad/s})t + (0.50 \text{ rad/s}^3)t^3$, with t in seconds.
- (a) Calculate the angular velocity of the spindle at $t = 3.0 \text{ s}$.
- (b) Calculate the angular acceleration of the spindle at $t = 3.0 \text{ s}$.
2. A paper mill reel has an angular acceleration $\alpha(t) = (8.0 \text{ rad/s}^2) - (2.0 \text{ rad/s}^3)t$. At $t = 0$ the reel turns at 3.0 rad/s and its angle is zero.
- (a) Find the angular velocity of the reel as a function of time.
- (b) Calculate the angle the reel turns through between $t = 0$ and $t = 3.0 \text{ s}$.
3. A turnstile arm is pushed and starts to turn clockwise at 5.0 rad/s . A steady counterclockwise angular acceleration of magnitude 2.0 rad/s^2 then acts on it. Take counterclockwise as positive.
- (a) Calculate the time at which the angular velocity of the arm is zero.
- (b) Calculate the angle of the arm at $t = 4.0 \text{ s}$, measured from its angle at $t = 0$.
4. A stepped pulley turns on a fixed axle. A cord on its inner step, radius 0.040 m , is drawn in, and the length taken in is $s(t) = (0.30 \text{ m/s}^2)t^2$. A second cord runs on the outer step, radius 0.110 m .
- (a) Calculate the angular velocity of the pulley at $t = 2.0 \text{ s}$.
- (b) Calculate the speed of the second cord at $t = 2.0 \text{ s}$.

5. A wind turbine blade tip is 34.0 m from the hub. The rotor starts from rest and speeds up at a steady 0.120 rad/s^2 .
- (a) Calculate the arc length the blade tip travels in the first 10.0 s.
 - (b) Calculate the magnitude of the total acceleration of the blade tip at $t = 10.0 \text{ s}$.
6. A salad spinner is cranked at a steady rate. Its basket has a radius of 0.115 m, and one seed sits against the inside wall of the basket. The basket makes 31 full turns in 5.00 s.
- (a) Calculate the arc length the seed travels in those 5.00 s.
 - (b) Calculate the speed of the seed while the basket is turning.
7. A cyclist pushes straight down on a pedal with a force of 210 N. The crank arm is 0.170 m long, and at this instant the crank arm points 40.0 degrees past the top of its circle, measured from straight up.
- (a) Calculate the lever arm of the push about the crank axle.
 - (b) Calculate the magnitude of the torque of the push about the crank axle.

8. A drum of radius 0.350 m turns on a fixed axle. Three ropes are wrapped around its rim and pulled, each tangent to the rim: 124 N counterclockwise, 86.0 N clockwise and 60.0 N counterclockwise. Take counterclockwise as positive.
- (a) Calculate the torque of the 86.0 N rope about the axle, with its sign.
 - (b) Calculate the net torque on the drum, with its sign.
9. A horizontal arm of length 2.0 m is pinned to a wall at its inner end. A load presses down on the arm along its whole length, with a force per unit length $w(x) = (150 \text{ N/m}^2)x$, where x is the distance from the pin in meters.
- (a) Calculate the total downward force the load exerts on the arm.
 - (b) Calculate the magnitude of the torque of that load about the pin.
10. A revolving door turns about a vertical axis. A torque of magnitude 38 N·m about that axis is needed to start it turning. A person pushes at right angles to one panel.
- (a) The person pushes with 55 N. Calculate the smallest distance from the axis at which that push will start the door turning.
 - (b) The person can reach only a point 0.50 m from the axis. Calculate the magnitude of the smallest force at that point that will start the door turning.
11. A tapered tent pole is treated as a thin rod 0.900 m long. Its mass for every meter of length grows along the pole as $\lambda(x) = (4.00 \text{ kg/m}^3)x^2$, where x is the distance in meters from the thin end.
- (a) Calculate the total mass of the pole.
 - (b) Calculate the rotational inertia of the pole about an axis through its thin end, at right angles to the pole.

12. A steel washer is an annular disk with a hole at its center. This one has an inner radius of 0.0200 m, an outer radius of 0.0500 m and a uniform areal density $\sigma = 42.0 \text{ kg/m}^2$. It turns about its central axis.
- (a) Calculate the mass of the washer.
 - (b) Integrate $r^2 dm$ over the washer to find its rotational inertia about that axis.
13. A thin circular hoop has a mass of 3.00 kg and a radius of 0.150 m. About the axis through its center, perpendicular to its plane, $I_{\text{cm}} = MR^2$. The hoop is then hung on a nail through the hoop itself, so the nail is 0.150 m from the center.
- (a) Calculate the rotational inertia of the hoop about the axis through its center.
 - (b) Calculate the rotational inertia of the hoop about a parallel axis through the nail, with the hoop turning in its own plane.
14. Three small balls of mass 0.300 kg, 0.500 kg and 0.800 kg are fixed at the three corners of a light triangular frame whose sides are each 0.600 m long.
- (a) Calculate the rotational inertia about an axis through the 0.800 kg ball, perpendicular to the plane of the frame.
 - (b) Calculate the rotational inertia about a parallel axis through the 0.300 kg ball instead.
15. A uniform diving board of mass 42.0 kg and length 3.40 m is bolted to the deck at its left end and rests on a roller 0.600 m from that end. A diver of mass 58.0 kg stands at the free end. The board stays level.
- (a) Calculate the magnitude of the upward force the roller exerts on the board.
 - (b) Calculate the magnitude of the force the bolt exerts on the board, and state its direction.

- 16.** The long horizontal arm of a tower crane is balanced about the mast on its own. A load of mass 1500 kg then hangs 18.0 m out along one side of the arm, and a counterweight of mass 4200 kg rides on the other side.
- Calculate how far from the mast the counterweight must sit for the arm to stay in rotational equilibrium.
 - The load is then run out to 24.0 m. Calculate how much farther from the mast the counterweight must move.
- 17.** A uniform flagpole of mass 26.5 kg and length 4.00 m is hinged at the ground and stands at 60.0 degrees to the ground. A horizontal wire runs from a wall to a point on the pole 3.00 m from the hinge and holds the pole in place.
- Calculate the magnitude of the torque of the weight of the pole about the hinge.
 - Calculate the tension in the wire.
- 18.** A pump impeller has a rotational inertia of $0.080 \text{ kg}\cdot\text{m}^2$ and is at rest at $t = 0$. From then on the net torque on it is $\tau(t) = (1.2 \text{ N}\cdot\text{m}) - (0.40 \text{ N}\cdot\text{m/s})t$.
- Calculate the angular acceleration of the impeller at $t = 0$.
 - Calculate the angular velocity of the impeller at $t = 3.0 \text{ s}$.
- 19.** A 5.00 kg block on a level frictionless table is joined by a light cord, over a pulley at the table edge, to a 3.00 kg block hanging freely. The pulley has radius 0.100 m, rotational inertia $0.0100 \text{ kg}\cdot\text{m}^2$ and a frictionless axle. The cord does not slip on the pulley.
- Calculate the magnitude of the acceleration of the blocks.
 - Calculate the tension in the section of cord attached to the hanging block.

20. A playground roundabout of radius 1.50 m is turning at 3.00 rad/s, with one child sitting at the rim. The child drags one foot along the ground. The drag acts on the rim as a steady tangential force of 44.0 N against the turning, and the roundabout stops in 5.00 s.
- (a) Calculate the magnitude of the angular acceleration that brings the roundabout to rest in that time.
 - (b) Calculate the rotational inertia of the roundabout and the child together about the axis.
- 21.* Two shafts in a machine are at the same angle at $t = 0$. Shaft A turns at a steady 12 rad/s. Shaft B starts from rest at $t = 0$ and speeds up at a steady 3.0 rad/s^2 . The two turn in the same direction.
- (a) Calculate the first instant after $t = 0$ at which the two shafts have turned through the same angle.
 - (b) Calculate the angle by which shaft B lags shaft A at the instant their angular velocities are equal.
- 22.* A bucket hangs from a rope wound on a drum of radius 0.045 m. A wheel of radius 0.12 m is fixed to the drum and turns with it. A belt drives that wheel from a motor pulley of radius 0.040 m and does not slip. The motor pulley turns with $\omega_m(t) = (8.0 \text{ rad/s}^2)t$.
- (a) Calculate the speed at which the bucket rises at $t = 3.0 \text{ s}$.
 - (b) Calculate the height the bucket rises during the first 3.0 s.

23.* A uniform disk of radius 0.400 m is cut from a sheet of areal density 12.0 kg/m^2 . A circular hole of radius 0.100 m is then cut out of it, centered 0.200 m from the center of the disk. For a solid disk, $I = \frac{1}{2}mR^2$ about its central axis.

- (a) Calculate the mass of the material removed to make the hole.

- (b) Calculate the rotational inertia of the drilled disk about the axis through the center of the original disk, perpendicular to its face.

24.* A uniform roller of mass 55.0 kg and radius 0.300 m rests on the floor against a step 0.120 m high. A horizontal force is applied at the top of the roller, 0.600 m above the floor, to begin lifting it over the step.

- (a) Calculate the horizontal distance from the edge of the step to the line along which the weight of the roller acts.

- (b) Calculate the magnitude of the smallest horizontal force at the top that begins to lift the roller over the step.

25.* A 6.00 kg crate hangs from a rope wound on the hub of a drum, of hub radius 0.100 m. The drum has a rotational inertia of $0.260 \text{ kg}\cdot\text{m}^2$ and turns on a frictionless axle. A brake pad presses on the rim, radius 0.250 m, with a tangential friction force of 9.00 N.

- (a) Calculate the magnitude of the acceleration of the crate as it descends.

- (b) Calculate the tension in the rope.

* Challenging.

AP Physics C: Mechanics, Equation Sheet

The constants, conventions and equations on this sheet are those the College Board provides on the AP Physics exam reference information.

CONSTANTS AND CONVERSION FACTORS

$$G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

universal gravitational constant

$$g = 9.8 \text{ m/s}^2$$

magnitude of the acceleration due to gravity at the surface of the Earth

$$g = 9.8 \text{ N/kg}$$

magnitude of the gravitational field strength at the surface of the Earth

CONVENTIONS

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT SYMBOLS

hertz	Hz
newton	N
joule	J
second	s
kilogram	kg
watt	W
meter	m

TRIGONOMETRIC FUNCTIONS AT COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

$$s = r\theta$$

Rectangular solid

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = a/c$$

$$\cos \theta = b/c$$

$$\tan \theta = a/b$$

KINEMATICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\Delta x = \int v_x(t) dt$$

$$\Delta v_x = \int a_x(t) dt$$

CENTER OF MASS

$$\vec{x}_{\text{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\vec{r}_{\text{cm}} = \frac{\int \vec{r} dm}{\int dm}$$

$$\lambda = \frac{d}{d\ell} m(\ell)$$

$$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

FORCE AND TRANSLATIONAL DYNAMICS

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

$$|\vec{F}_g| = G \frac{m_1 m_2}{r^2}$$

$$|\vec{F}_f| \leq |\mu \vec{F}_N|$$

$$\vec{F}_s = -k\Delta\vec{x}$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

$$T = \frac{1}{f}$$

WORK, ENERGY AND POWER

$$K = \frac{1}{2}mv^2$$

$$W = \int_a^b \vec{F} \cdot d\vec{r}$$

$$\Delta K = \sum W_i = \sum F_{\parallel, i} d_i$$

$$\Delta U = - \int_a^b \vec{F}_{\text{cf}}(r) \cdot d\vec{r}$$

$$F_x = -\frac{dU(x)}{dx}$$

$$U_s = \frac{1}{2}k(\Delta x)^2$$

$$U_G = -G\frac{m_1 m_2}{r}$$

$$\Delta U_g = mg\Delta y$$

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = \frac{dW}{dt}$$

LINEAR MOMENTUM

$$\vec{p} = m\vec{v}$$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}(t) dt = \Delta \vec{p}$$

TORQUE AND ROTATIONAL DYNAMICS

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$v = r\omega$$

$$a_T = r\alpha$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$I_{\text{tot}} = \sum I_i = \sum m_i r_i^2$$

$$I = \int r^2 dm$$

$$I' = I_{\text{cm}} + Md^2$$

$$\alpha_{\text{sys}} = \frac{\Sigma \tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

ENERGY AND MOMENTUM OF ROTATING SYSTEMS

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$W = \int \tau \cdot d\theta$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$\Delta L = \int \tau dt$$

$$\Delta x_{\text{cm}} = r\Delta\theta$$

OSCILLATIONS

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_{\text{phys}} = 2\pi\sqrt{\frac{I}{mgd}}$$

$$x = x_{\text{max}} \cos(\omega t + \phi)$$

VECTORS

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

$$\vec{r} = (A\hat{i} + B\hat{j} + C\hat{k})$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax)$$

IDENTITIES

$$\log(a \cdot b^x) = \log a + x \log b$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

WHAT THE LETTERS MEAN

a	acceleration
d	distance
E	energy
f	frequency
F	force
I	rotational inertia
J	impulse
k	spring constant
K	kinetic energy
ℓ	length
L	angular momentum
m	mass
M	mass
p	momentum
P	power
r	radius, distance, or position
t	time
T	period
U	potential energy
v	velocity or speed
W	work
x	position or distance
y	vertical position
α	angular acceleration
θ	angular position
λ	linear mass density
μ	coefficient of friction
τ	torque
ϕ	phase angle
ω	angular frequency or angular speed