

Torque and Rotational Dynamics Practice Quiz**Name:** _____**Date:** _____

$$g = 10 \text{ m/s}^2$$

Part A. Multiple Choice

1. A windmill blade turns counterclockwise and is slowing at a steady rate. Counterclockwise is taken as positive. The angular velocity and the angular acceleration of the blade are
 - A. a negative angular velocity and a negative angular acceleration
 - B. a positive angular velocity and a negative angular acceleration
 - C. a positive angular velocity and a positive angular acceleration
 - D. a negative angular velocity and a positive angular acceleration
2. A drill bit spins at a steady 40 rad/s. The time it needs to sweep through 8.0 rad is
 - A. 5.0 s
 - B. 320 s
 - C. 1.26 s
 - D. 0.20 s
3. A centrifuge rotor slows at a steady rate from 90 rad/s to 30 rad/s while it turns through 120 rad. Take the sense of the spin as positive. Its angular acceleration is
 - A. -30 rad/s^2
 - B. -60 rad/s^2
 - C. -0.50 rad/s^2
 - D. $+30 \text{ rad/s}^2$
4. For a turning disk, the graph of angular velocity against time is a straight line. It starts at a positive value and reaches zero at $t = 4.0 \text{ s}$. During those first 4.0 s the disk
 - A. turns at a steady angular velocity right up until it stops at $t = 4.0 \text{ s}$
 - B. has an angular acceleration that falls away to zero as the graph reaches the axis
 - C. turns in one sense the whole time, with the angle it has swept still growing
 - D. reverses the sense of its turning at some instant before $t = 4.0 \text{ s}$
5. Two wheels start from rest, each with a steady angular acceleration. The first has twice the angular acceleration of the second. Each is checked once it has turned through the same angle. At that moment,
 - A. the two wheels have the same angular velocity and they reached that angle at the same time
 - B. the first wheel has the greater angular velocity, and it reached that angle in less time
 - C. the two wheels have the same angular velocity, because they turned through the same angle
 - D. the second wheel has the greater angular velocity, because it took longer to get there
6. A circular saw blade of radius 0.090 m spins at 300 rad/s. The speed of one tooth at the edge of the blade is
 - A. 54 m/s
 - B. 3300 m/s
 - C. 300 m/s
 - D. 27 m/s
7. A winch drum of radius 0.25 m winds in cable at a steady 8.0 rad/s and lifts a crate. The time the crate needs to rise 6.0 m is
 - A. 0.75 s
 - B. 0.19 s

- C. 3.0 s
D. 1.5 s
8. A bead is clamped to a spoke of a wheel that turns about its axle. For that bead, the rule that its linear speed equals its distance from the axle times the angular velocity holds only while the bead
- A. stays put at one place along the spoke as the wheel turns
 - B. sits out at the rim rather than part way along the spoke
 - C. turns with a steady angular velocity and no angular acceleration
 - D. sweeps through an angle that is measured in degrees, not radians
9. On a turning platform a point 0.60 m from the axis moves at 3.0 m/s. A second point on the same platform moves at 1.0 m/s. That second point sits a distance from the axis of
- A. 0.60 m
 - B. 0.20 m
 - C. 1.8 m
 - D. 0.40 m
10. A disk slows at a steady rate. Follow one point on its rim as the disk slows. The two parts of the acceleration of that point behave so that
- A. the centripetal part holds a fixed magnitude while the tangential part falls off
 - B. both parts fall off together and hold a fixed ratio to each other throughout
 - C. the tangential part turns around to point toward the axis once the disk slows
 - D. the tangential part holds a fixed magnitude while the centripetal part falls off
11. A child pushes perpendicular to a revolving door panel with a force of 40 N, at a point 0.80 m from the central shaft. The torque about the shaft is
- A. 32 N·m
 - B. 40 N·m
 - C. 50 N·m
 - D. 0.020 N·m
12. A horizontal bar turns freely about a pivot at its left end. A force pushes straight down on the bar at its right end. Counterclockwise is taken as positive. The torque of that force about the pivot is
- A. zero, because the force is perpendicular to the bar
 - B. zero, because the pivot holds that end of the bar in place
 - C. negative, because acting alone it would turn the bar clockwise
 - D. positive, because the force acts right at the far end of the bar
13. A cyclist pushes straight down on a bicycle pedal with a force of 150 N. The crank arm is 0.18 m long and makes an angle of 60 degrees with the vertical. The torque about the crank axle is
- A. 46.8 N·m
 - B. 23.4 N·m
 - C. 27.0 N·m
 - D. 13.5 N·m
14. A rope pulls on a gate at a point 2.0 m from the hinge, in a direction the gardener may choose. Together with the magnitude of the pull, the one quantity that fixes the torque about the hinge is
- A. the distance from the hinge to the point of contact, whatever way the rope points
 - B. the full length of the gate, measured from the hinge out to its far edge
 - C. the angle the rope makes with the gate, with no distance needed at all
 - D. the perpendicular distance from the hinge to the line of the rope

15. One force acts on a bar at a single point. The axis that torques are taken about is then moved to a new place. Moving the axis
- A. changes the torque only while the new axis lies somewhere along the bar itself
 - B. reverses the sign of the torque for every new axis that might be chosen
 - C. changes the torque, and the torque is zero for any axis lying on the line of the force
 - D. leaves the torque unchanged, because the force acting on the bar has not changed at all
16. A 0.50 kg ball sits at the far end of a light arm, 0.80 m out from the axle it spins around. Its rotational inertia about that axle is
- A. $0.32 \text{ kg}\cdot\text{m}^2$
 - B. $0.40 \text{ kg}\cdot\text{m}^2$
 - C. $0.64 \text{ kg}\cdot\text{m}^2$
 - D. $0.16 \text{ kg}\cdot\text{m}^2$
17. Two small masses are fixed to a light frame that turns about a fixed axis: 3.0 kg at 0.20 m from the axis and 1.0 kg at 0.60 m from the axis. The rotational inertia of the pair is
- A. $1.4 \text{ kg}\cdot\text{m}^2$
 - B. $0.48 \text{ kg}\cdot\text{m}^2$
 - C. $1.2 \text{ kg}\cdot\text{m}^2$
 - D. $0.16 \text{ kg}\cdot\text{m}^2$
18. Two children of equal mass sit on opposite sides of a light turning platform, each the same distance from the axis. One child then moves in closer to the axis. The rotational inertia of the whole system
- A. falls in direct proportion to the distance that one child has moved in
 - B. rises, because the mass is no longer placed evenly around the axis
 - C. falls, because the distance of that child from the axis is squared in the sum
 - D. does not change, because no mass was added to the system and none was taken off it
19. A thin hoop of mass 1.6 kg and radius 0.30 m has $I_{\text{cm}} = MR^2$ about its central axis. The hoop now hangs on a nail through one point of the hoop and swings in its own plane. About the nail, its rotational inertia is
- A. $0.14 \text{ kg}\cdot\text{m}^2$
 - B. $0.22 \text{ kg}\cdot\text{m}^2$
 - C. $0.58 \text{ kg}\cdot\text{m}^2$
 - D. $0.29 \text{ kg}\cdot\text{m}^2$
20. A rigid system is turned about two parallel axes, one through its center of mass and one a short distance away. Set against the value about the center of mass, the rotational inertia about the other axis is
- A. greater, by the mass of the system times the square of the distance between the axes
 - B. greater, by the mass of the whole system times the distance between the two parallel axes
 - C. the same, because none of the mass of the system has been moved anywhere at all
 - D. less, because the second axis sits closer to part of the mass of the system
21. A wheel on good bearings turns at a steady angular velocity, with nothing driving it. Which statement about the wheel is true?
- A. The torques on it add to a steady value that is not zero while it turns.
 - B. It is in rotational equilibrium, because the torques on it add to zero.
 - C. It is not in rotational equilibrium, because it is turning rather than at rest.
 - D. It is in rotational equilibrium only at the instant the wheel comes to rest.
22. A light rod hangs level from a string tied at its center. A 0.60 kg block hangs 0.20 m to the left of the string. A 0.40 kg block is hung on the right side. The rod hangs level when that block is
- A. 0.20 m from the string

- B. 0.12 m from the string
 - C. 0.30 m from the string
 - D. 0.13 m from the string
23. A flat plate rests on frictionless ice. One steady push is applied along a line that passes through the center of mass of the plate. While that push acts, the plate is
- A. in translational equilibrium, because the push acts through the center of mass
 - B. in both kinds of equilibrium, because only one push is applied to the plate
 - C. in neither kind of equilibrium, because any push at all must start the plate spinning
 - D. in rotational equilibrium, because the net torque about the center of mass is zero
24. Two brackets hold a uniform shelf of mass 4.0 kg that is 1.20 m long, one bracket at the left end and one 0.90 m to the right of it. A 6.0 kg box stands 0.50 m in from the left end. The left bracket pushes up with
- A. 27 N
 - B. 40 N
 - C. 50 N
 - D. 60 N
25. Three forces act on a bar. The net force on the bar is zero, and the net torque about one chosen axis is zero as well. The net torque about any other axis is
- A. zero as well, and that follows from the net force on the bar being zero
 - B. zero only for another axis that also lies somewhere along the bar itself
 - C. able to take any value at all, since a torque is always tied to its axis
 - D. zero only for an axis that runs through the center of mass of the bar
26. A revolving stage has a rotational inertia of $18 \text{ kg}\cdot\text{m}^2$ about its central axis. A net torque of $27 \text{ N}\cdot\text{m}$ acts on it. The angular acceleration of the stage is
- A. 0.67 rad/s^2
 - B. 486 rad/s^2
 - C. 2.7 rad/s^2
 - D. 1.5 rad/s^2
27. A turnstile of rotational inertia $2.4 \text{ kg}\cdot\text{m}^2$ is pushed with 30 N perpendicular to an arm, 0.80 m from the axis. Friction opposes the turning with a torque of $6.0 \text{ N}\cdot\text{m}$. The angular acceleration is
- A. 12.5 rad/s^2
 - B. 18 rad/s^2
 - C. 7.5 rad/s^2
 - D. 10 rad/s^2
28. A wheel is turning counterclockwise. A net clockwise torque is then applied to it. Over the next short interval the wheel
- A. keeps turning counterclockwise and its angular velocity falls off
 - B. turns clockwise at once, following the sense of the new net torque
 - C. keeps the angular velocity it had, since the torque opposes the spin
 - D. stops at that very instant, because the torque opposes the spin
29. A flywheel of rotational inertia $0.80 \text{ kg}\cdot\text{m}^2$ starts from rest. A steady net torque of $2.0 \text{ N}\cdot\text{m}$ acts on it for 6.0 s. The angle it turns through in that time is
- A. 2.5 rad
 - B. 45 rad
 - C. 90 rad
 - D. 15 rad

30. A wheel turns on an axle, and a small pulley is fixed to the same axle. A rope is pulled with the same force, first around the rim of the wheel and then around the small pulley. The angular acceleration is
- A. the same in the two cases, because the force and the turning system are the same in both
 - B. greater with the rope on the small pulley, because the force then acts closer in to the axle
 - C. greater with the rope at the rim, because the rotational inertia itself falls away as the force moves out
 - D. greater with the rope at the rim, because the torque is greater and the rotational inertia is the same
- 31.* A spin dryer basket speeds up from rest at 2.0 rad/s^2 for 5.0 s , then slows at a steady 1.0 rad/s^2 until it stops. Its angular velocity on reaching half of its total angle is
- A. 7.1 rad/s
 - B. 10 rad/s
 - C. 8.7 rad/s
 - D. 5.0 rad/s
- 32.* A flywheel is coming to a stop at a steady rate. One spot sits on its rim and a second half way in toward the axle. At one instant, the angle between the total acceleration of a spot and its own radius is
- A. the same for the two spots, because the ratio of the two parts does not depend on the radius
 - B. greater for the spot on the rim, because that spot travels faster along its own circular path
 - C. greater for the inner spot, because its centripetal part is the weaker of its own two parts
 - D. impossible to compare until the mass of the flywheel and the rate of the slowing are given
- 33.* A student claims that the rotational inertia of a rigid body about an axis through its center of mass is less than its value about any other axis whatsoever. That claim
- A. fails, because the theorem takes a term away for an axis that runs inside the body
 - B. fails, because an axis in a new direction through the center of mass can give less
 - C. holds, because the parallel axis theorem adds a term that is never a negative one
 - D. holds, because the center of mass is the one point the mass of the body balances about
- 34.* A uniform diving board of mass 30 kg and length 4.0 m is bolted to the deck at its back end and rests on a roller 1.0 m from that end. A 50 kg swimmer stands at the free end. The force from the bolt is
- A. 1500 N , directed downward
 - B. 800 N , directed upward
 - C. 2600 N , directed upward
 - D. 1800 N , directed downward
- 35.* A uniform 20 kg gate of width 1.2 m is at rest on vertical hinges, with $I = \frac{1}{3}ML^2$ about that line. A child pushes the outer edge with 25 N , always perpendicular to it. After a 90 degree swing the angular velocity is
- A. 1.8 rad/s
 - B. 4.4 rad/s
 - C. 3.1 rad/s
 - D. 24 rad/s

* Challenging.

Part B. Reasoning

1. For a rigid body turning about a fixed axis, explain why the angle is used as the position variable rather than the distance that one point of the body travels.
2. A ceiling fan is switched off and slows to rest at a steady rate. Explain why the fan turns through a greater angle during the first half of the slowing time than during the second half.
3. Explain how a wheel can have an angular velocity of zero at one instant and an angular acceleration that is not zero at that same instant. Describe a case in which this happens.
4. One wheel is reported at $+4.0$ rad/s and another at -4.0 rad/s. State what the two wheels have in common and what differs between them, and explain what fixes the sign.
5. On a carousel, one horse is mounted near the axis and another out near the rail. Compare the angular velocities of the two horses and compare their linear speeds, and explain both comparisons.
6. A garden hose is wound onto a reel. Explain why the length of hose taken up in one full turn is fixed by the radius of the coil and not by how fast the reel is turned.
7. Explain why the angle has to be measured in radians before the relation between arc length and angle, or the relation between linear speed and angular velocity, may be used.

8. Describe why the steering wheel of a truck without power steering is built with a large radius, and explain the comparison with a wheel of small radius in terms of torque about the steering column.
9. A meter stick is balanced on a pivot at its middle. One hand presses straight down on the left end and the other hand presses straight down on the right end. Explain why the two torques about the pivot carry opposite signs.
10. A child pulls on the outer rail of a playground merry-go-round, with the rope at 50 degrees to the line from the axis out to her hands. Explain which part of that pull turns the merry-go-round and what the other part does.
11. A batter slides her hands farther up the handle of a bat, toward the heavy barrel, and swings about her hands. Explain what that does to the rotational inertia of the bat about the axis through her hands.
12. Weights are clamped at several places along a light arm that spins about a fixed axle. Describe how to build up the rotational inertia of the whole group, and explain why a weight clamped close to the axle adds little to it.
13. A door can turn about its hinge line, and the same door can be spun about a line through its center. Explain why one body does not have a single rotational inertia.
14. A tall cabinet stands on a level floor and is pushed gently near its top so that it leans. Explain, in terms of torques about the front bottom edge, why the cabinet tips over once the line of its weight passes beyond that edge.

15. Explain why a tower crane carries a heavy counterweight on the arm opposite its load, and state what has to be true of the torques about the mast while a load hangs from the crane.
16. Describe how to pick the axis for a torque equation in an equilibrium problem so that one unknown force drops out, and explain why picking it that way is allowed.
17. A cart is pulled along a level floor by a cord that winds onto a winch drum turning on a fixed axle. Describe the two bodies that have to be treated separately here, and state the one equation that ties them together.
18. Two bicycle wheels of equal mass are spun up to the same angular velocity and left to slow under the same friction torque at the axle. One wheel carries most of its mass out at the rim. Explain which wheel takes longer to stop.
19. Several torques act on a flywheel at once. Describe how to find its angular acceleration, and explain why the torques have to be added with their signs before anything is divided.
20. Two flywheels start from rest on their axles and are driven by the same net torque for the same length of time. One has twice the rotational inertia of the other. Compare the angles they turn through, and explain the comparison.
- 21.* A car is fitted with tires of greater radius and nothing else is changed. The speedometer is driven by the angular velocity of a wheel. Explain whether the speedometer now reads above or below the true road speed.

- 22.* A student claims that a force applied farther out from the axis always produces a torque of greater magnitude than the same force applied closer in. Explain the flaw in that claim.
- 23.* A uniform rod has a known rotational inertia about an axis at one end. Somebody finds the value about a parallel axis a short distance beyond that end by adding the mass times the square of that short distance. Explain the flaw.
- 24.* A uniform bar is pinned to a wall at one end and held level by a cable running from its far end up to a higher point on the wall. A student claims the force the pin exerts has to point along the bar. Explain the flaw.
- 25.* A cord on the rim of a pulley is pulled by a hand with a steady 8 N. In a second trial a block of weight 8 N hangs from the same cord instead. A student claims the angular acceleration is the same in both trials. Explain the flaw.

* Challenging.

Part C. Problems

1. A helicopter rotor is turning at 42 rad/s when the engine is shut off. The rotor then slows at a steady rate and comes to rest after 70 full turns. Take the direction the rotor turns as positive.
 - (a) Calculate the angle the rotor turns through before it comes to rest, in radians.

 - (b) Calculate the angular acceleration of the rotor while it slows.

2. A roulette wheel is spun and starts at 8.0 rad/s . Friction then slows it at a steady 0.40 rad/s^2 until it stops. Take the direction of the spin as positive.
 - (a) Calculate the time the wheel takes to stop.

 - (b) Calculate the angle the wheel turns through in that time.

3. A paint mixer shaft is turning at 3.0 rad/s when the motor is set higher. The shaft then speeds up at a steady 1.5 rad/s^2 for 6.0 s.
 - (a) Calculate the angular velocity of the shaft at the end of the 6.0 s.

 - (b) Calculate the angle the shaft turns through during those 6.0 s.

4. A fish runs with the line of a fishing reel and pulls it off the spool at a steady 2.4 m/s . The line leaves the edge of the spool, which has a radius of 0.040 m.
 - (a) Calculate the angular velocity of the spool.

 - (b) Calculate the number of turns the spool makes in 3.0 s.

5. A ferris wheel of radius 16 m starts from rest and speeds up at a steady rate, reaching an angular velocity of 0.25 rad/s after 40 s.
- (a) Calculate the angular acceleration of the wheel.

 - (b) Calculate the tangential acceleration of a seat on the rim while the wheel speeds up.
6. A small gear of radius 0.030 m has its teeth meshed with those of a large gear of radius 0.105 m. The teeth do not slip. The small gear turns at 14 rad/s.
- (a) Calculate the angular velocity of the large gear.

 - (b) Calculate the number of turns the large gear makes while the small gear makes 7 turns.
7. A round faucet handle of radius 0.035 m is turned by two fingers. Each finger pushes along the rim with 6.0 N. The two pushes act at opposite ends of a diameter, in opposite directions, so both turn the handle the same way.
- (a) Calculate the torque of one finger about the center of the handle.

 - (b) Calculate the net torque on the handle from both fingers.

8. A hand pushes on the lid of a car trunk at a point 0.70 m from the hinge line. The push has a magnitude of 90 N and is aimed at 35 degrees to the lid.
- (a) Calculate the component of the push that lies at right angles to the lid.
 - (b) Calculate the torque of the push about the hinge line.
9. A spool turns on a fixed axle. Both ropes on it pull at right angles to their radii. A rope on its hub, radius 0.080 m, is pulled with 40 N and turns the spool counterclockwise. A rope on its rim, radius 0.200 m, is pulled with 12 N and turns it clockwise. Counterclockwise is positive.
- (a) Calculate the net torque on the spool.
 - (b) Calculate the pull on the rim rope that would make the net torque zero, with the hub rope unchanged.
10. Four small masses, each 0.50 kg, are bolted at the corners of a light square frame of side 0.60 m. The frame turns about an axis through its center, at right angles to the frame.
- (a) Calculate the distance from the axis to one corner of the frame.
 - (b) Calculate the rotational inertia of the four masses about that axis.
11. A steel bar is uniform, 1.50 m long, and has a mass of 6.0 kg. For an axis through the center of the bar, at right angles to it, $I_{\text{cm}} = \frac{1}{12}ML^2$.
- (a) Calculate the rotational inertia of the bar about that central axis.
 - (b) Calculate the rotational inertia about a parallel axis 0.50 m from the center.

12. A light ring of radius 0.25 m carries six small bolts, each of mass 0.40 kg, spaced evenly around its rim. The ring turns about an axis through its center, at right angles to the ring.
- (a) Calculate the rotational inertia of the six bolts about that axis.

 - (b) Two of the bolts are moved in to a radius of 0.125 m. Calculate the new rotational inertia.
13. A light beam 1.20 m long is pivoted at a point 0.10 m from its left end. A bag of apples of mass 3.0 kg hangs from the left end. A sliding weight of mass 0.50 kg hangs from the beam on the right of the pivot. Use $g = 10 \text{ m/s}^2$.
- (a) Calculate the magnitude of the torque of the weight of the bag about the pivot.

 - (b) Calculate how far from the pivot the sliding weight must hang for the beam to balance.
14. A uniform pipe of mass 30 kg and length 4.0 m is carried level by two workers. One holds it at the left end, and the other holds it at a point 1.0 m from the right end. Use $g = 10 \text{ m/s}^2$.
- (a) Calculate the upward force from the worker at the left end.

 - (b) Calculate the upward force from the other worker.

15. A forearm is held level. The biceps pulls straight up on it at a point 0.040 m from the elbow joint. A 5.0 kg dumbbell is held in the hand, 0.32 m from the joint. The weight of the forearm is small enough to leave out. Use $g = 10 \text{ m/s}^2$.
- (a) Calculate the force the biceps exerts on the forearm.
 - (b) Calculate the magnitude of the force the joint exerts on the forearm.
16. A uniform bench top of mass 12 kg and length 2.0 m rests on two legs, each 0.30 m in from an end. A child of mass 25 kg sits on the left end. Use $g = 10 \text{ m/s}^2$.
- (a) Calculate the upward force from the right leg.
 - (b) Calculate the mass of a child on the left end that would just lift the right leg off the floor.
17. A cement mixer drum has a rotational inertia of $2.5 \text{ kg}\cdot\text{m}^2$ about its axis. It starts from rest, and a steady net torque of $7.5 \text{ N}\cdot\text{m}$ acts on it.
- (a) Calculate the angular acceleration of the drum.
 - (b) Calculate the angular velocity of the drum after 4.0 s.

18. A ship is steered with a wheel of rotational inertia $1.8 \text{ kg}\cdot\text{m}^2$, whose handles stand 0.45 m from the axis. A sailor pushes tangentially on one handle with 24 N . Friction in the mounting gives a steady opposing torque of $1.2 \text{ N}\cdot\text{m}$.

(a) Calculate the net torque on the wheel.

(b) Calculate the angular acceleration of the wheel.

19. A washing machine drum is turning at 24 rad/s when the brake is applied. The brake brings the drum to rest in 8.0 s at a steady rate. The drum has a rotational inertia of $0.75 \text{ kg}\cdot\text{m}^2$. Take the direction of the turning as positive.

(a) Calculate the angular acceleration of the drum while it slows.

(b) Calculate the magnitude of the braking torque.

20. A cable drum of radius 0.20 m turns on a fixed axle and has a rotational inertia of $0.40 \text{ kg}\cdot\text{m}^2$. A motor applies a steady torque of $12 \text{ N}\cdot\text{m}$ to the drum. The cable runs to a 5.0 kg sled on frictionless ice and does not slip.

(a) Calculate the acceleration of the sled.

(b) Calculate the tension in the cable.

21.* A telescope dome starts from rest. It speeds up at a steady rate for 4.0 s, turns at that angular velocity for 10 s, then slows at a steady rate to rest in 6.0 s. In all it turns through 30 rad.

(a) Calculate the angular velocity the dome holds during the middle stage.

(b) Calculate the angular acceleration of the dome during the final stage.

22.* A ski lift cable runs over a drive wheel of radius 0.60 m and does not slip. The wheel starts from rest, speeds up at a steady 0.50 rad/s^2 for 8.0 s, and then holds that angular velocity.

(a) Calculate the speed of the cable once the wheel holds its angular velocity.

(b) Calculate the distance a chair travels during the first 20 s.

23.* Three disks, each of mass 0.80 kg and radius 0.10 m, are bolted to a light arm so that none of them can turn about its own center. Their centers sit 0.20 m, 0.40 m and 0.60 m from the axle. For one disk, $I_{\text{cm}} = \frac{1}{2}MR^2$.

(a) Calculate the rotational inertia about the axle of the disk whose center is 0.40 m out.

(b) Calculate the total rotational inertia of the three disks about the axle.

24.* An awning arm 2.0 m long is pinned to a wall at its inner end and held level by a stay. The stay runs from the wall to a point 1.60 m out along the arm, at 40 degrees to the arm. The arm is uniform, of mass 10 kg, and a 15 kg lamp hangs at its tip. Use $g = 10 \text{ m/s}^2$.

(a) Calculate the tension in the stay.

(b) Calculate the magnitude of the force the pin exerts on the arm.

25.* A cord is wound on the hub of a flywheel at rest. The hub radius is 0.10 m, and $I = 0.50 \text{ kg}\cdot\text{m}^2$. The cord is pulled with a steady 25 N until 4.0 m of it has come off, and it then leaves the hub. Axle friction opposes the turning with a steady 0.50 N·m.

(a) Calculate the angular velocity of the flywheel at the moment the cord leaves the hub.

(b) Calculate the further angle the flywheel turns through before it comes to rest.

* Challenging.

AP Physics 1, Equation Sheet

The constants, conventions and equations on this sheet are those the College Board provides on the AP Physics exam reference information.

CONSTANTS AND CONVERSION FACTORS

$$G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

universal gravitational constant

$$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5 \text{ Pa}$$

one atmosphere of pressure

$$g = 9.8 \text{ m/s}^2$$

magnitude of the acceleration due to gravity at the surface of the Earth

$$g = 9.8 \text{ N/kg}$$

magnitude of the gravitational field strength at the surface of the Earth

CONVENTIONS

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.
- Fluids are assumed to be ideal, and pipes are assumed to be completely filled by fluid, unless otherwise stated.

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	<i>T</i>
10^9	giga	<i>G</i>
10^6	mega	<i>M</i>
10^3	kilo	<i>k</i>
10^{-2}	centi	<i>c</i>
10^{-3}	milli	<i>m</i>
10^{-6}	micro	μ
10^{-9}	nano	<i>n</i>
10^{-12}	pico	<i>p</i>

UNIT SYMBOLS

hertz	Hz
newton	N
joule	J
pascal	Pa
kilogram	kg
second	s
meter	m
watt	W

TRIGONOMETRIC FUNCTIONS AT COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

$$s = r\theta$$

Rectangular solid

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = a/c$$

$$\cos \theta = b/c$$

$$\tan \theta = a/b$$

KINEMATICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

CENTER OF MASS

$$\vec{x}_{\text{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

FORCE AND TRANSLATIONAL DYNAMICS

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$$

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t} = m \vec{a}$$

$$|\vec{F}_g| = G \frac{m_1 m_2}{r^2}$$

$$|\vec{F}_f| \leq |\mu \vec{F}_N|$$

$$\vec{F}_s = -k \Delta \vec{x}$$

$$a_c = \frac{v^2}{r}$$

WORK, ENERGY AND POWER

$$K = \frac{1}{2}mv^2$$

$$W = F_{\parallel} d = F d \cos \theta$$

$$\Delta K = \sum W_i = \sum F_{\parallel, i} d_i$$

$$U_s = \frac{1}{2}k(\Delta x)^2$$

$$U_G = -\frac{Gm_1m_2}{r}$$

$$\Delta U_g = mg\Delta y$$

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = F_{\parallel}v = Fv \cos \theta$$

LINEAR MOMENTUM

$$\vec{p} = m\vec{v}$$

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$$

TORQUE AND ROTATIONAL DYNAMICS

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$v = r\omega$$

$$a_T = r\alpha$$

$$\tau = r_{\perp}F = rF \sin \theta$$

$$I = \sum m_i r_i^2$$

$$I' = I_{\text{cm}} + Md^2$$

$$\alpha_{\text{sys}} = \frac{\Sigma \tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

ENERGY AND MOMENTUM OF ROTATING SYSTEMS

$$K = \frac{1}{2}I\omega^2$$

$$W = \tau\Delta\theta$$

$$L = I\omega$$

$$L = rmv \sin \theta$$

$$\Delta L = \tau\Delta t$$

$$\Delta x_{\text{cm}} = r\Delta\theta$$

OSCILLATIONS

$$T = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$x = A \cos(2\pi ft)$$

$$x = A \sin(2\pi ft)$$

FLUIDS

$$\rho = \frac{m}{V}$$

$$P = \frac{F_{\perp}}{A}$$

$$P = P_0 + \rho gh$$

$$P_{\text{gauge}} = \rho gh$$

$$F_b = \rho Vg$$

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

WHAT THE LETTERS MEAN

<i>a</i>	acceleration
<i>A</i>	amplitude or area
<i>d</i>	distance
<i>E</i>	energy
<i>f</i>	frequency
<i>F</i>	force
<i>h</i>	height
<i>I</i>	rotational inertia
<i>J</i>	impulse
<i>k</i>	spring constant
<i>K</i>	kinetic energy
<i>ℓ</i>	length
<i>L</i>	angular momentum
<i>m</i>	mass
<i>M</i>	mass
<i>p</i>	momentum
<i>P</i>	power or pressure
<i>r</i>	radius or distance
<i>t</i>	time
<i>T</i>	period
<i>U</i>	potential energy
<i>v</i>	velocity or speed
<i>V</i>	volume
<i>W</i>	work
<i>x</i>	position
<i>y</i>	vertical position
<i>α</i>	angular acceleration
<i>θ</i>	angle or angular position
<i>μ</i>	coefficient of friction
<i>ρ</i>	density
<i>τ</i>	torque
<i>ω</i>	angular speed