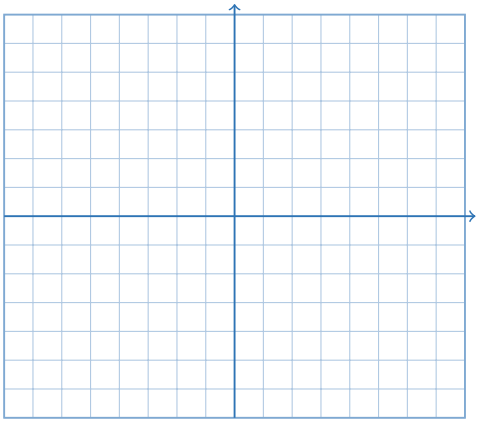
Problem Set 4: Work, Energy, and Power

Name: _____ **Date:** _____

Topics 3.1 through 3.5. Take $\vec{a}_g = -9.80665\,\hat{j}$ m/s^2. Name the system before you write an energy term, and say where you are putting the zero of potential energy. Out of 100 points.

1. A car of mass 1200 kg travels east along a straight level road at 15.0 m/s. A truck travels east along the same road at a steady 20.0 m/s. (3.1) **[17 pts]**



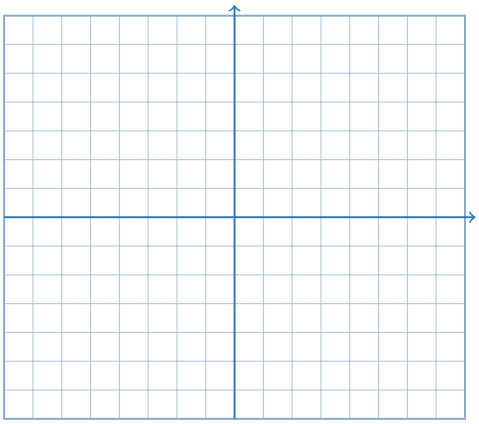
(a) Find the translational kinetic energy of the car as measured by an observer standing at the roadside. **[4 pts]**

(b) Find the translational kinetic energy of the car as measured by an observer riding in the truck, and say which of your two answers is the kinetic energy of the car. **[4 pts]**

(c) On the grid, to a stated scale, sketch the kinetic energy of the car against its speed for speeds from 0 to 30 m/s as the roadside observer measures them. **[4 pts]**

(d) The car now speeds up steadily from 15.0 m/s to 20.0 m/s. Find the change in its kinetic energy as measured from the roadside and as measured from the truck. The two answers do not even share a sign. Write a short argument reconciling that with the work-energy theorem, given that the same force acts in both accounts. **[5 pts]**

2. A particle of mass 2.00 kg moves along the x axis. The only force on it is $F_x(x) = (6.00\ \mathrm{N/m^2})x^2 - (12.0\ \mathrm{N/m})x$. At $x = 0$ the particle is moving in the $+x$ direction at 4.00 m/s. (3.2) **[17 pts]**



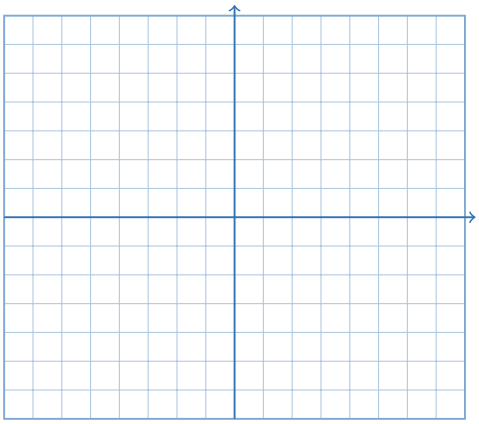
(a) Find the work done by this force on the particle from $x = 0$ to $x = 4.00$ m. **[4 pts]**

(b) Find the speed of the particle at $x = 4.00$ m. **[4 pts]**

(c) Find the position at which the particle is moving most slowly on the way out, and say whether it ever comes to rest before $x = 4.00$ m. Support the answer, do not assert it. **[4 pts]**

(d) A second particle is carried along the straight path from the origin to the point $(3.00\ \mathrm{m}, 4.00\ \mathrm{m})$ while a constant force $\vec{F} = (5.00\ \mathrm{N})\hat{\imath} + (2.00\ \mathrm{N})\hat{\jmath}$ acts on it. Find the work done by that force using the dot product, and then the angle between the force and the displacement. **[5 pts]**

3. A single conservative force acts on a particle moving along the x axis: $F_x(x) = -(8.00\ \mathrm{N/m})x + (3.00\ \mathrm{N/m^3})x^3$. (3.3) **[16 pts]**



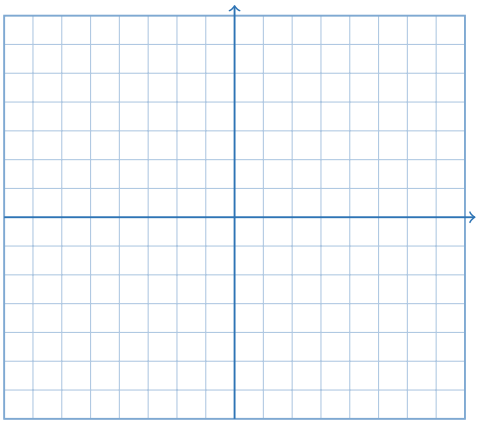
(a) Find the potential energy function $U(x)$ of the system, taking $U = 0$ at $x = 0$. **[4 pts]**

(b) Differentiate your $U(x)$ and show that it returns the force you were given. **[4 pts]**

(c) Find every position at which the system is in equilibrium. **[4 pts]**

(d) Classify each equilibrium position as stable or unstable, and justify each one from what the force does to a small displacement, not only from the shape of the graph. **[4 pts]**

4. A particle of mass 2.00 kg moves along the x axis in a system whose potential energy is $U(x) = (2.00\ \mathrm{J/m^2})x^2 - (0.500\ \mathrm{J/m^4})x^4$, valid for $-2.00\ \mathrm{m} \le x \le 2.00$ m. (3.3) **[17 pts]**



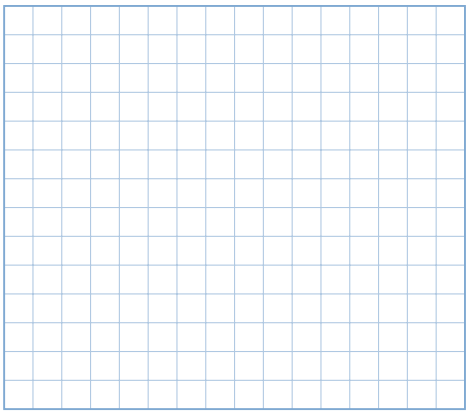
(a) Sketch $U(x)$ on the grid to a stated scale over the whole interval. **[4 pts]**

(b) The particle is released from rest at $x = 0.500$ m. Find its speed as it passes $x = 0$. **[4 pts]**

(c) The particle is instead given a total mechanical energy of 1.80 J and set moving near the origin. Find the turning points of that motion. **[4 pts]**

(d) Find the largest total mechanical energy for which a particle set moving near the origin stays trapped there, and say what happens to a particle given more than that. Mark the trapping energy on your sketch. **[5 pts]**

5. A block of mass 2.50 kg is released from rest at the top of a rough ramp whose top is 1.60 m above the bottom, and arrives at the bottom moving at 4.20 m/s. It then slides along a smooth horizontal surface into a spring of stiffness 900 N/m. (3.4) **[16 pts]**



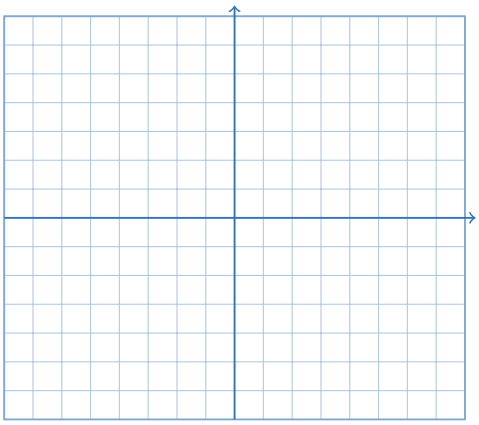
(a) On the grid, draw energy bar charts for the system of the block, the ramp, the spring and the Earth at three moments: at the top, at the bottom of the ramp, and at greatest compression. **[4 pts]**

(b) Find the mechanical energy dissipated on the ramp, and name where it went. **[4 pts]**

(c) Find the greatest compression of the spring. **[4 pts]**

(d) Write a short argument, in prose, explaining how one slide can be described by a system whose total energy is constant and by a system whose energy is not, with neither description being wrong. **[4 pts]**

6. A car of mass 1400 kg starts from rest on a level road. From $t = 0$ to $t = 8.00$ s the engine delivers power to the car according to $P(t) = (4.50 \times 10^4\ \mathrm{W/s})\,t$. Friction and air resistance are small enough to ignore over this interval. (3.5) **[17 pts]**



(a) Find the total work done on the car in the first 8.00 s. **[4 pts]**

(b) Find the speed of the car at $t = 8.00$ s. **[4 pts]**

(c) Derive an expression for the speed of the car as a function of time over this interval, state the speed at $t = 4.00$ s, and sketch the speed against time on the grid. **[4 pts]**

(d) Find the instantaneous power at $t = 8.00$ s and the average power over the interval. Then show that the force driving the car is constant over this interval, and explain how a constant force can deliver a power that grows with time. **[5 pts]**