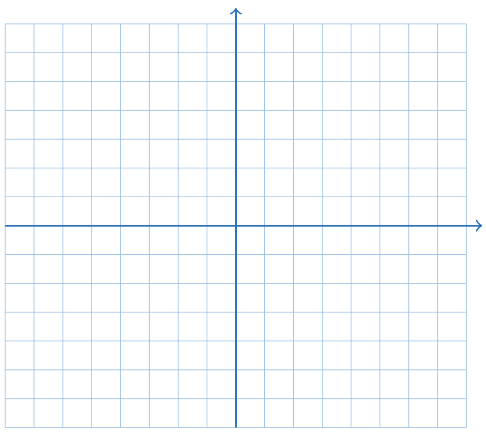
Problem Set 2: Force and Translational Dynamics

Name: ______________________________ **Date:** ______________

Topics 2.1 through 2.5. Every surface is frictionless unless otherwise specified. Take $\vec{a}_g = -9.80665\,\hat{\jmath}$ m/s^2. Out of 100 points.

1. Three small objects lie in the xy-plane: 1.00 kg at the origin, 2.00 kg at $(3.00$ m, 4.00 m$)$, and 3.00 kg at $(6.00$ m, $0)$. **[17 pts]**



(a) Find the center of mass of the system as a position vector in component form, and plot the three objects and that point on the grid to a stated scale. **[4 pts]**

(b) A thin rod of length L lies along the x-axis from 0 to L with linear mass density $\lambda(x) = \lambda_0(1 + x/L)$. Find x_{cm} by integration. **[4 pts]**

(c) State the one feature of a uniform rod that lets you write down its center of mass without any integral, and say why the rod in (b) does not have it. **[4 pts]**

(d) The rod of (b) is cut at $x = L/2$. Find the center of mass and the mass of each piece, then show that replacing the two pieces by point masses at those locations reproduces your answer to (b). State the general principle this illustrates. **[5 pts]**

2. A 4.00 kg block rests on a frictionless ramp inclined at $30.0°$ to the horizontal. It is held in place by a rope that is *horizontal*, not parallel to the ramp surface. **[17 pts]**

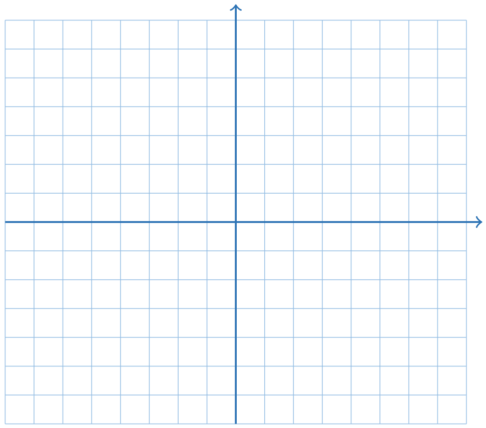
(a) Draw the free-body diagram of the block on the grid, and state which way your axes point and why that choice beats the obvious one here. **[4 pts]**

(b) Find the tension in the rope and the magnitude of the normal force, symbolically first and then as numbers. **[4 pts]**

(c) Show that your normal force also equals $mg\cos\theta + F_T \sin\theta$, and explain in one sentence why the two expressions must agree. **[4 pts]**

(d) The rope is slowly rotated from horizontal until it lies along the ramp surface, the block staying at rest throughout. Write the tension as a function of the rope angle ϕ measured from the ramp surface, find the angle at which the tension is least, and give that least tension. **[5 pts]**

3. A uniform rope of mass 2.40 kg and length 3.00 m hangs at rest from a ceiling. Let y be measured upward from the free lower end. **[16 pts]**



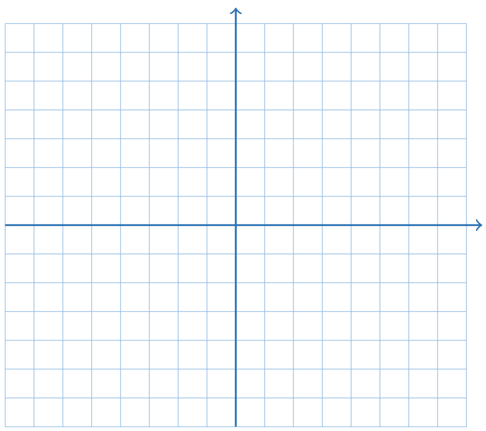
(a) Find the linear mass density of the rope. **[4 pts]**

(b) By drawing a free-body diagram of the portion of rope below height y, show that the tension there is $F_T(y) = \lambda g y$. **[4 pts]**

(c) Evaluate the tension halfway up and at the ceiling, and check the second value against the weight of the whole rope. Then state what assumption makes an ideal string have the same tension everywhere, and where in (b) it would enter. **[4 pts]**

(d) A 1.00 kg block is hung from the free lower end. Write the new $F_T(y)$, sketch it against y on the grid, and find the height at which the tension is twice its value at the lower end. **[4 pts]**

4. A 2.00 kg cart starts from rest on a straight frictionless track at $t = 0$. The only horizontal force on it is $F(t) = (6.00t)$ N, directed along the track, with t in seconds. **[17 pts]**



(a) Write Newton's second law for this cart as a differential equation and find $a(t)$. **[4 pts]**

(b) Find $v(t)$ and $x(t)$ by integration, stating the limits you used and what each constant of integration means physically. **[4 pts]**

(c) Evaluate the speed and displacement at $t = 4.00$ s, and sketch v against t on the grid. **[4 pts]**

(d) Explain why the constant-acceleration equations cannot be used here, naming the assumption they rest on. Then find the single constant force that would carry the cart over the same displacement in the same 4.00 s, and explain why it does not also give the same final speed. **[5 pts]**

5. A light, inextensible string passes over an ideal pulley. A 5.00 kg block hangs from one end and a 3.00 kg block from the other. The system is released from rest. **[16 pts]**

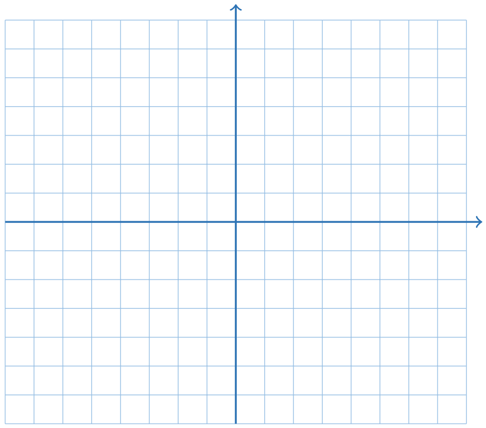
(a) Draw a free-body diagram for each block on the grid, stating your positive direction for each. **[4 pts]**

(b) Find the magnitude of the acceleration of the blocks. **[4 pts]**

(c) Find the tension in the string, and verify it using the block you did not use to find it. **[4 pts]**

(d) Find the acceleration of the center of mass of the two-block system, and show that the net external force on that system equals the total mass times that acceleration. Identify the external forces explicitly. **[4 pts]**

6. A 60.0 kg person stands on a bathroom scale in an elevator. Starting from rest at $t = 0$, the elevator's upward speed rises steadily to 8.00 m/s over 4.00 s, holds at 8.00 m/s until $t = 9.00$ s, then falls steadily to rest at $t = 11.0$ s. **[17 pts]**



(a) Sketch the elevator's velocity against time on the grid, labeling both axes with quantity and unit. **[4 pts]**

(b) Draw the free-body diagram of the person and write Newton's second law for the vertical direction. **[4 pts]**

(c) Find the scale reading, in newtons, in each of the three intervals, and find the acceleration that would make the scale read zero. **[4 pts]**

(d) The person's mass never changed. Explain what did change, and why a scale does not measure mass. Then suppose the elevator reaches 8.00 m/s in 2.00 s instead, the rest of the trip unchanged: find the new peak reading and the new total rise, and name one quantity the change leaves untouched. **[5 pts]**