Problem Set 1: Kinematics

Name: ______________________________ **Date:** ____________________

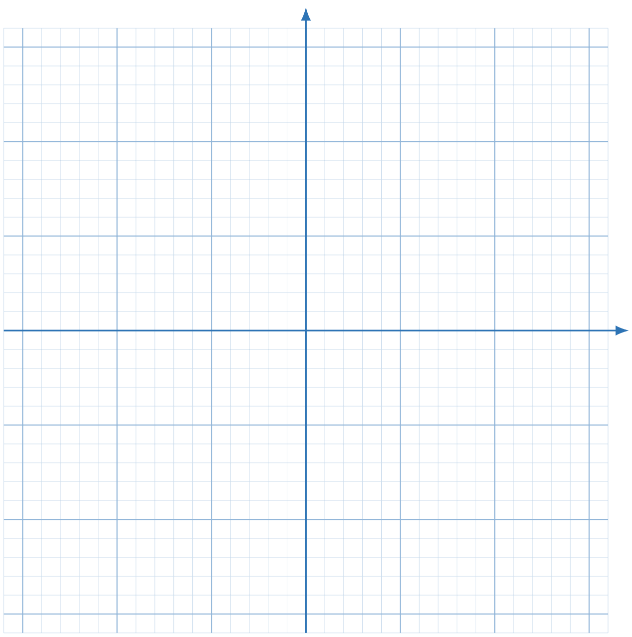
This set covers the whole of Unit 1. Work symbolically wherever you can, and substitute once at the end. Units belong in every answer, and calculus changes them: a derivative divides by the units of the variable you differentiate with respect to, an integral multiplies by them. Every definite integral needs limits, and the limits come from the situation. Position is $\vec{r} = x\hat{\imath} + y\hat{\jmath}$. Use $g = 10\ \mathrm{m/s^2}$. Where a sketch is asked for, a grid sits at the left of the solution space with room to work beside it; label the axes yourself and choose your own scale. Graded out of **100 points**, marked beside each number. **Extensions** are optional extra credit, up to **12 points**.

1. **[12 pts]**

(a) Classify each as a scalar or a vector: distance, speed, position, displacement, velocity, acceleration, mass, elapsed time.

(b) For $\vec{A} = 6\hat{\imath} + 8\hat{\jmath}$, find $|\vec{A}|$ and the unit vector $\hat{A}$. Verify that $|\hat{A}| = 1$.

(c) With $\vec{B} = 9\hat{\imath} - 12\hat{\jmath}$, find $\vec{C} = \vec{A} + \vec{B}$ and $|\vec{C}|$. Draw all three on the grid, tip to tail, and explain why $|\vec{C}|$ is not $|\vec{A}| + |\vec{B}|$.



(d) A particle confined to the x-axis has velocity -6.0 m/s. State its speed and its direction of travel, and explain why no arrow notation is needed to describe direction in one dimension.

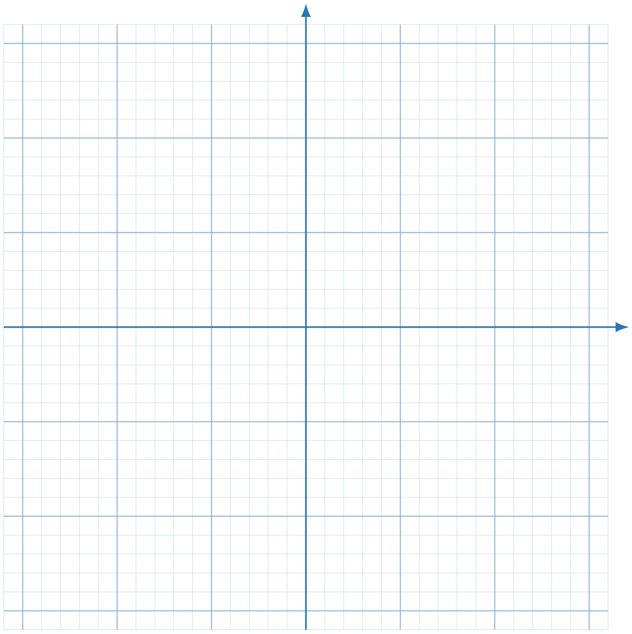
Extension (optional). Show that $\hat{A} = \vec{A}/|\vec{A}|$ has magnitude 1 for *any* nonzero $\vec{A}$, working from components rather than from this particular case. **[+1.5 pts]**

2. **[12 pts]**

A drone's position changes from $\vec{r}_0 = -3\hat{\imath} + 4\hat{\jmath}$ m to $\vec{r} = 9\hat{\imath} - \hat{\jmath}$ m.

(a) Find $\Delta\vec{r}$ in component form and its magnitude.

(b) Plot $\vec{r}_0$, $\vec{r}$ and $\Delta\vec{r}$ on the grid, drawing $\Delta\vec{r}$ from tip to tip.



(c) The drone did not travel in a straight line. State what you can and cannot determine about the distance it flew, given only the two positions.

(d) Modeling the drone treats it as a single point with mass, ignoring its size, shape and spinning rotors. Name one question about this drone for which that model would be inadequate, and say what it discards that matters there.

Extension (optional). Write the integral that gives the distance flown, in terms of $\vec{v}(t)$, and say why the two endpoints alone can never determine it. **[+1.5 pts]**

3. **[14 pts]**

A particle moves with $\vec{r}(t) = (4t^2 - 3t)\hat{\imath} + 2t^3\hat{\jmath}$, in meters with t in seconds.

(a) Find the average velocity from $t = 0$ to $t = 2.0$ s.

(b) Find $\vec{v}(t)$ and $\vec{a}(t)$ by differentiation.

(c) Evaluate $\vec{v}$ and $\vec{a}$ at $t = 2.0$ s, giving the magnitude of each with units.

(d) Compare your answers to (a) and (c), and explain why they differ. Then state what would have to be true of the motion for them to agree.

(e) Find the instant at which $v_x = 0$. Is the particle at rest then? Justify your answer.

Extension (optional). Compute the average velocity over $t = 2.0$ to $t = 2.01$ s and compare it with $\vec{v}(2.0)$. Say what this demonstrates about averages taken over very small intervals. **[+1.5 pts]**

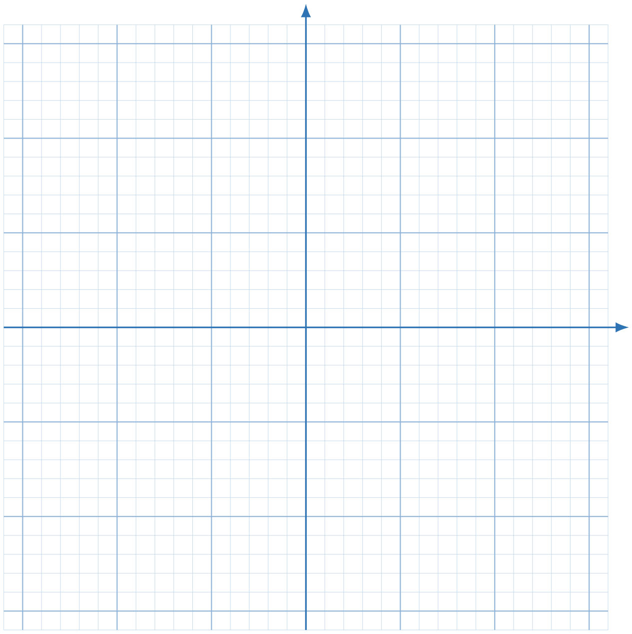
4. **[12 pts]**

A car rounds a bend at a constant speed of 20 m/s. Its velocity changes from due east to due north over an interval of 8.0 s. Take east as $+\hat{\imath}$ and north as $+\hat{\jmath}$.

(a) Write $\vec{v}_0$ and $\vec{v}$ in unit-vector notation, and find $\Delta\vec{v}$.

(b) Find the average acceleration in component form, its magnitude, and its direction measured counterclockwise from the $+x$-axis.

(c) Draw $\vec{v}_0$, $\vec{v}$ and $\Delta\vec{v}$ on the grid.



(d) The car's speed never changed. Explain how it can nevertheless have been accelerating throughout, and state precisely what was changing.

Extension (optional). If the bend were a quarter circle traversed at constant speed, the magnitude of the *instantaneous* acceleration would be v^2/r. Find the radius of that circle, using the fact that the car covers a quarter of its circumference in 8.0 s. **[+1.5 pts]**

5. **[14 pts]**

A stone is thrown vertically upward at 15 m/s from the top of a building 20 m tall. Take upward as positive and the ground as $y = 0$.

(a) Write $y(t)$ and $v_y(t)$ using the constant-acceleration equations.

(b) Find the time at which the stone reaches its highest point, and the height above the ground it reaches.

(c) Find the time at which the stone strikes the ground.

(d) Find its speed on impact. Then obtain the same answer a second way, using $v^2 = v_0^2 + 2a\Delta y$, and confirm the two agree.

Extension (optional). Derive $v^2 = v_0^2 + 2a\Delta y$ from the other two constant-acceleration equations by eliminating t. State where in the derivation you rely on a being constant. **[+1.5 pts]**

6. **[14 pts]**

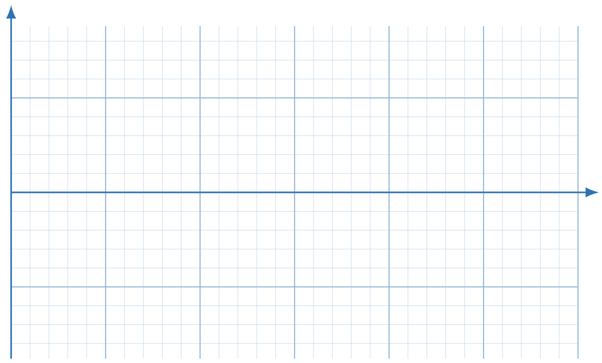
A particle starts at the origin with $\vec{v}_0 = (2.0\ \mathrm{m/s})\hat{\imath}$ and has acceleration $\vec{a}(t) = (4.0\ \mathrm{m/s^2})\hat{\imath} + (6.0t\ \mathrm{m/s^3})\hat{\jmath}$.

(a) Find $\vec{v}(t)$ by integration, naming each constant of integration physically.

(b) Find $\vec{r}(t)$.

(c) Evaluate $\vec{v}$ and $\vec{r}$ at $t = 3.0$ s.

(d) Sketch v_x against t for $0 \le t \le 3.0$ s and shade the area representing the x-displacement. Confirm that the shaded area matches the x-component of your answer to (c).



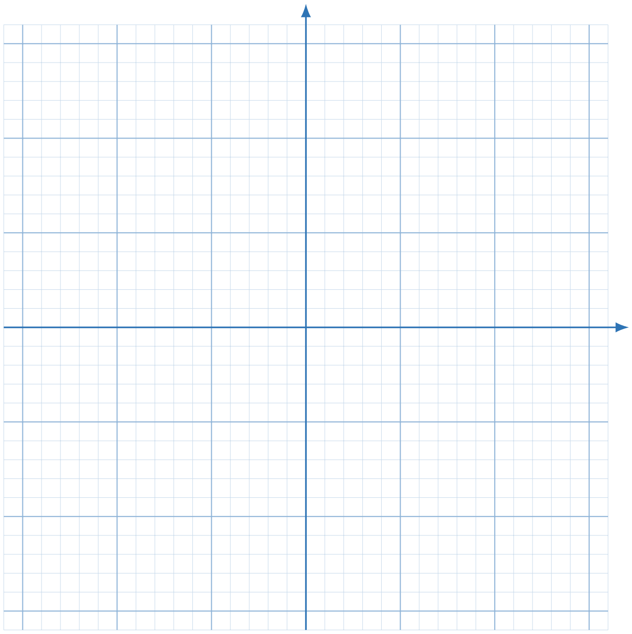
(e) Explain why the constant-acceleration equations of Problem 5 cannot be used for the y-motion here.

Extension (optional). Find the instant at which the particle's velocity makes an angle of $45°$ with the x-axis. **[+1.5 pts]**

7. **[10 pts]**

A boat's engine drives it at 5.0 m/s due north relative to the water. The current runs at 3.0 m/s due east relative to the ground. The river is 120 m wide.

(a) Find the boat's velocity relative to the ground, as a magnitude and a direction, and draw the three vectors tip to tail on the grid.



(b) Find how long the crossing takes and how far downstream the boat lands. Explain why the current does not change the crossing time.

(c) An observer drifting with the current and an observer on the bank each measure the boat's velocity and its acceleration. State what each measures, and explain why they disagree about one and agree about the other.

Extension (optional). The current is not uniform: $v_c(y) = 3.0 \sin(\pi y/120)$ m/s, with y measured from the near bank. Set up, but do not evaluate, the integral for the total downstream drift. **[+1.5 pts]**

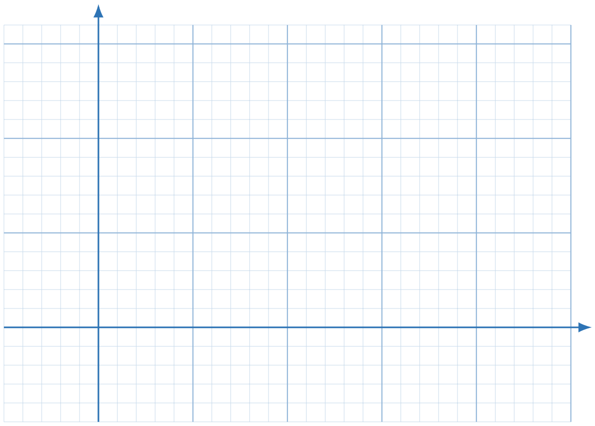
8. **[12 pts]**

A projectile is launched from the origin with $\vec{v}_0 = (24\ \mathrm{m/s})\hat{\imath} + (18\ \mathrm{m/s})\hat{\jmath}$, with $\vec{a} = -g\hat{\jmath}$ throughout.

(a) Find $|\vec{v}_0|$ and the launch angle above the horizontal.

(b) Find the time to maximum height, the maximum height, the total flight time and the range.

(c) Sketch the trajectory on the grid, marking the apex and the landing point.



(d) The horizontal launch speed is now doubled, the vertical component unchanged. State what happens to the flight time and to the range, and explain what this shows about the two components of the motion.

Extension (optional). Eliminate t between $x(t)$ and $y(t)$ to show the path is a parabola $y = \alpha x - \beta x^2$, giving α and β in terms of v_{0x}, v_{0y} and g. **[+1.5 pts]**