Practice Test: Kinematics, Forces and Translational Dynamics

Name: ______________________________ **Date:** ______________

Equations. $\vec{v} = \dfrac{d\vec{r}}{dt}$ $\vec{a} = \dfrac{d\vec{v}}{dt}$ $\vec{r}_{cm} = \dfrac{1}{M}\int \vec{r}\,dm$ $M = \int dm$ $dm = \lambda\,dx$ $\sum \vec{F} = m\dfrac{d\vec{v}}{dt}$ $\sum \vec{F}_{ext} = M\vec{a}_{cm}$ $\vec{F}_{AB} = -\vec{F}_{BA}$ $F_g = G\dfrac{m_1 m_2}{r^2}$ $|\vec{g}| = \dfrac{GM}{r^2}$ $|\vec{F}_{f,k}| = \mu_k|\vec{F}_n|$ $|\vec{F}_{f,s}| \leq \mu_s|\vec{F}_n|$ $\vec{F}_s = -k\Delta\vec{x}$ $\vec{F}_r = -b\vec{v}$ $a_c = \dfrac{v^2}{r}$ $T = \dfrac{2\pi r}{v}$

Given. $\vec{a}_g = -9.80665\,\hat{\jmath}$ m/s^2, which may be taken as 10 N/kg $G = 6.67 \times 10^{-11}$ N·m^2/kg^2 Air resistance is negligible unless a question says otherwise.

Section I. Multiple choice.

Only 15 multiple choice questions will be required on the real test next week. Suggested time: 30 minutes.

1. A particle moves along the x-axis with $x(t) = \left(4t^3 - 6t^2\right)$ m, with t in seconds. Its acceleration at $t = 1.0$ s is **[3 pts]**

(A) zero. (B) -12 m/s^2.
(C) 12 m/s^2. (D) 24 m/s^2.

2. For a particle moving along a line, the area between the acceleration against time graph and the time axis, over some interval, gives **[3 pts]**

(A) the change in velocity over that interval.
(B) the displacement over that interval.
(C) the change in speed over that interval.
(D) the average acceleration over that interval.

3. Two vectors each have magnitude 6.0. One points $30°$ above the $+x$-axis and the other $30°$ below it. The magnitude of their sum is **[3 pts]**

(A) 12.0. (B) 6.0.
(C) 8.5. (D) 10.4.

4. A particle's velocity against time is a single straight line with negative slope, crossing zero at $t = 4.0$ s. Over the interval from $t = 0$ to $t = 8.0$ s, its displacement is **[3 pts]**

(A) equal in magnitude to the total distance it traveled.
(B) zero.
(C) negative, and equal in magnitude to the distance it traveled.
(D) impossible to determine without knowing the slope.

5. Frame S' moves at constant velocity relative to frame S. The acceleration of a particle measured in S', compared with its acceleration measured in S, is **[3 pts]**

(A) the same.
(B) larger by the relative speed of the frames.
(C) smaller by the relative speed of the frames.
(D) different, unless the particle is at rest.

6. A projectile is launched from level ground with speed v_0 at an angle θ above the horizontal. When it returns to its launch height, its speed is **[3 pts]**

(A) zero. (B) $v_0 \cos\theta$.
(C) v_0. (D) $v_0 \sin\theta$.

7. A thin rod of length L lies along the x-axis from 0 to L with linear mass density $\lambda(x) = \lambda_0\left(1 - \dfrac{x}{L}\right)$. Its center of mass is at **[3 pts]**

(A) $x = \dfrac{L}{2}$. (B) $x = \dfrac{L}{3}$.
(C) $x = \dfrac{2L}{3}$. (D) $x = \dfrac{L}{4}$.

8. A block of mass m slides down a rough incline of angle θ at constant velocity. The magnitude of the friction force exerted on it is **[3 pts]**

(A) $\mu_k mg$.
(B) zero, since the block is not accelerating.
(C) mg.
(D) $mg\sin\theta$.

9. Block A is pulled along a frictionless floor by a horizontal string of non-negligible mass, which trails behind it to block B. The whole arrangement accelerates. The tension in the string **[3 pts]**

(A) is greatest where the string meets block A.
(B) is the same at every point along the string.
(C) is greatest where the string meets block B.
(D) is zero at both ends and greatest in the middle.

10. A 2.0 kg object moves along the x-axis with $v(t) = \left(4t^2 - 2t\right)$ m/s, with t in seconds. The magnitude of the net force exerted on it at $t = 1.5$ s is **[3 pts]**

(A) 6.0 N.
(B) 12 N.
(C) 20 N.
(D) 10 N.

11. The gravitational field strength at the surface of a uniform solid sphere of radius R is g. At a distance $\frac{R}{4}$ from its center, the field strength is **[3 pts]**

(A) $16g$.
(B) $\frac{g}{4}$.
(C) $4g$.
(D) $\frac{g}{16}$.

12. A block rests on an incline whose angle is slowly increased. At the angle θ at which the block is just about to slip, **[3 pts]**

(A) $\tan\theta = \mu_k$.
(B) $\sin\theta = \mu_s$.
(C) $\cos\theta = \mu_s$.
(D) $\tan\theta = \mu_s$.

13. Two identical ideal springs, each of force constant k, are joined end to end in series and a block is hung from the lower one. The effective force constant of the pair is **[3 pts]**

(A) $\frac{k}{2}$.
(B) $2k$.
(C) k.
(D) $\frac{k}{4}$.

14. An object is released from rest in a fluid that exerts a resistive force $\vec{F}_r = -b\vec{v}$. As it falls, the magnitude of its acceleration **[3 pts]**

(A) stays constant at g.
(B) starts at zero and grows toward g.
(C) starts at g and falls toward zero.
(D) is zero throughout, since it reaches terminal speed at once.

15. A car rounds a frictionless curve of radius r, banked at angle θ, at the one speed for which no friction is needed. That speed satisfies **[3 pts]**

(A) $v^2 = gr\sin\theta$.
(B) $v^2 = gr\tan\theta$.
(C) $v^2 = gr\cos\theta$.
(D) $v^2 = gr$.

16. A particle moves along a line with $v(t) = \left(6t - t^2\right)$ m/s, with t in seconds and $t \geq 0$. It reverses direction at **[3 pts]**

(A) $t = 3.0$ s.
(B) $t = 0$ only.
(C) it never reverses direction.
(D) $t = 6.0$ s.

17. A particle moves along a line with velocity $v(t)$. Its displacement between $t = 0$ and $t = T$ is **[3 pts]**

(A) $\displaystyle\int_0^T v\,dt$.
(B) $\displaystyle\int_0^T |v|\,dt$.
(C) $v(T) - v(0)$.
(D) $\dfrac{dv}{dt}$ evaluated at $t = T$.

18. Ignoring air resistance, the path $y(x)$ of a projectile launched at an angle to the horizontal is **[3 pts]**

(A) a straight line.
(B) a circular arc.
(C) a parabola.
(D) a hyperbola.

19. A ball is released from rest inside a train that is moving along a straight track at constant velocity. As seen by a passenger on the train, the ball falls **[3 pts]**

(A) straight down.
(B) backward, toward the rear of the train.
(C) forward, toward the front of the train.
(D) along a parabola, curving toward the rear.

20. Car A travels due east at 20 m/s and car B travels due north at 15 m/s, both measured relative to the ground. The velocity of car A relative to car B is **[3 pts]**

(A) 5.0 m/s, directed east.
(B) 35 m/s, directed north of east.
(C) 25 m/s, directed north of east.
(D) 25 m/s, directed south of east.

21. For a system of particles acted on by external forces, the center of mass moves as though **[3 pts]**

(A) it carried the mass of the heaviest particle in the system.
(B) all of the system's mass were concentrated there and every external force acted at that point.
(C) no forces acted on it at all.
(D) it were attached to the heaviest particle.

22. A uniform rope of total mass m, length ℓ and linear density λ hangs from a ceiling with nothing attached to its lower end. The tension a distance y above the lower end is **[3 pts]**

(A) mg.
(B) mgy.
(C) zero.
(D) λgy.

23. A 2.0 kg object starts from rest and has acceleration $a(t) = 3t\ \mathrm{m/s^2}$, with t in seconds. Its velocity at $t = 2.0$ s is **[3 pts]**

(A) 6.0 m/s.
(B) 3.0 m/s.
(C) 12 m/s.
(D) 1.5 m/s.

24. The magnitude of the gravitational field at the exact center of a uniform solid sphere is **[3 pts]**

(A) infinite, because the distance is zero.
(B) equal to its value at the surface.
(C) zero.
(D) half its value at the surface.

25. A block is held against a vertical wall by a horizontal force pressing it into the wall. The block does not slide. The magnitude of the friction force exerted on the block by the wall is **[3 pts]**

(A) μ_s times the horizontal force.
(B) equal to the weight of the block.
(C) zero, because the block is not moving.
(D) μ_k times the horizontal force.

26. Two identical ideal springs, each of force constant k, are mounted side by side so that both stretch by the same amount when a load is applied. The effective force constant of the pair is **[3 pts]**

(A) $\frac{k}{2}$.
(B) k.
(C) $\frac{k}{4}$.
(D) $2k$.

27. An object released from rest falls through a fluid that exerts a resistive force $\vec{F}_r = -b\vec{v}$. It reaches terminal speed when **[3 pts]**

(A) the magnitude of the resistive force equals the magnitude of the weight.
(B) its velocity falls to zero.
(C) its acceleration reaches g.
(D) the resistive force falls to zero.

28. An object travels in a circle at constant speed. The net force exerted on it is **[3 pts]**

(A) tangent to the circle, in the direction of motion.
(B) zero, since the speed is not changing.
(C) directed toward the center of the circle.
(D) directed away from the center of the circle.

29. A satellite moves in a circular orbit of radius R about a body of mass M. Its orbital speed is **[3 pts]**

(A) $\sqrt{GMR}$.
(B) $\sqrt{\frac{GM}{R}}$.
(C) $\frac{GM}{R}$.
(D) $\frac{GM}{R^2}$.

30. An object moves in a straight line at constant velocity. Which statement must be true? **[3 pts]**

(A) No forces are exerted on it.
(B) Exactly one force is exerted on it.
(C) Every force exerted on it is vertical.
(D) The vector sum of the forces exerted on it is zero.

Section II. Free response.

Two free-response questions will be required on the real test next week. Show all your work. Include all units when applicable.

Question 1. **Mathematical Routines** **25 pts, suggested time 18 minutes**

A small sphere of mass m is released from rest in a fluid. Besides the gravitational force, the fluid exerts on the sphere a resistive force $\vec{F}_r = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the sphere. Buoyancy is negligible. Take the downward direction as positive.

(a) Write the differential equation that governs the speed v of the sphere as it falls. Begin by writing a fundamental physics principle or an equation from the reference information.

(b) Derive an expression for the terminal speed v_T of the sphere in terms of m, b and physical constants.

(c) Solve the differential equation from part (a) for v as a function of time, subject to the sphere being released from rest. Express your answer in terms of m, b, t and physical constants.

(d) The sphere has mass 0.40 kg and the constant b is $0.50\ \mathrm{kg/s}$.

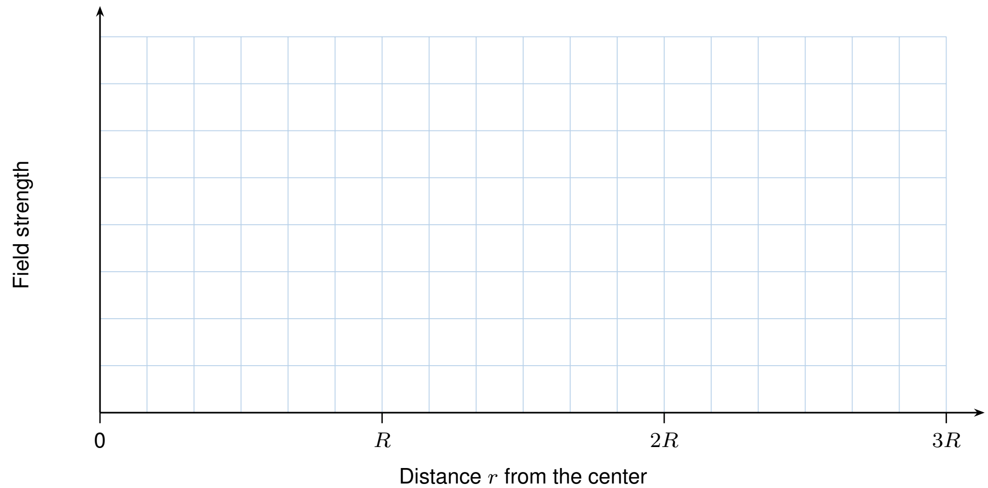
(i) Calculate the terminal speed of the sphere.

(ii) Calculate the time at which the sphere is moving at 90 percent of its terminal speed.

Question 2. **Translation Between Representations** **30 pts, suggested time 22 minutes**

A planet may be modeled as a uniform solid sphere of mass M and radius R. Let g_s be the magnitude of the gravitational field at its surface.

(a) On the axes below, sketch the magnitude of the gravitational field as a function of the distance r from the center of the planet, from $r = 0$ out to $r = 3R$. Mark the value at $r = R$ on the vertical axis.



(b) Derive an expression for the magnitude of the gravitational field inside the planet, at a distance $r < R$ from its center, in terms of M, R, r and physical constants. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

(c) Write the expression for the field outside the planet, and show that your two expressions agree at $r = R$. State what that agreement tells you about the graph you drew in part (a).

(d) Indicate whether the magnitude of the gravitational field at $r = \dfrac{R}{2}$ is greater than, less than, or equal to its magnitude at $r = 2R$. Justify your answer. In your justification, include qualitative reasoning beyond mathematical derivations or expressions.

Question 3. **Experimental Design and Analysis** **25 pts, suggested time 18 minutes**

A group of students is asked to measure the acceleration due to gravity using an Atwood machine: two hangers connected by a light string over a low friction pulley. They may move known masses from one hanger to the other, which changes the difference between the two hanging masses while keeping the total the same. They also have a meterstick and a motion sensor that reports the acceleration of a hanger directly.

(a)

(i) Indicate the quantities that could be measured, and describe a procedure that would allow the students to determine g using a linear graph.

(ii) Briefly describe a method for reducing the experimental uncertainty in the measured quantities.

(b)

(i) Indicate what quantities should be graphed on the horizontal and on the vertical axes so that the graph is linear. Clearly state which quantity goes on each axis.

(ii) Describe how g could be found from a feature of that graph.

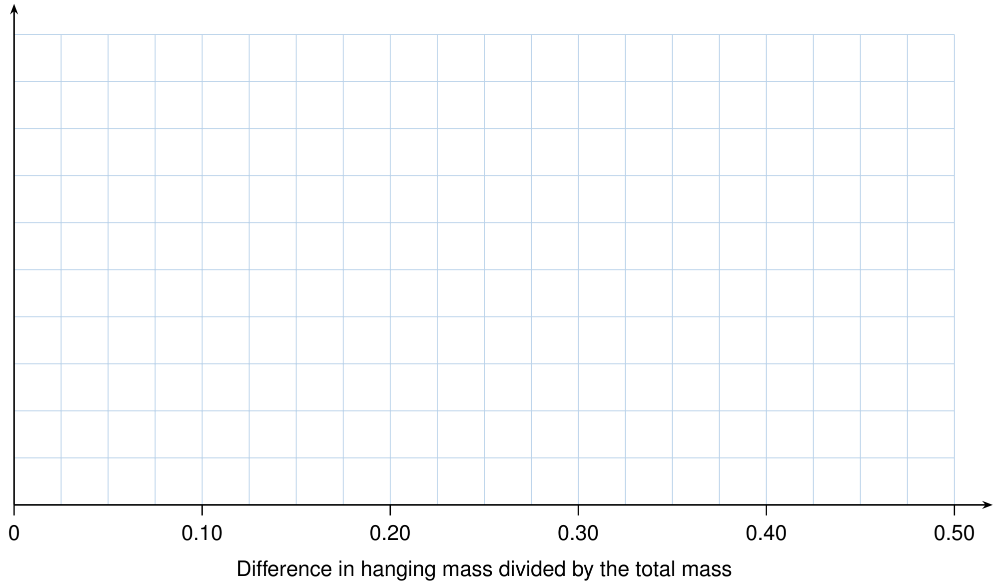
(c) With the total mass held at 1.00 kg, the students obtain the data below.

Mass difference divided by total mass	0.10	0.20	0.30	0.40	0.50
Measured acceleration (m/s^2)	1.05	1.90	2.95	3.85	4.95

(i) Label the vertical axis of the graph below with a quantity and a numerical scale.

(ii) Plot the data points.

(iii) Draw a straight best-fit line through the plotted points.



(d) Using the best-fit line you drew in part (c)(iii), calculate an experimental value for g.

Question 4. Qualitative/Quantitative Translation **20 pts, suggested time 14 minutes**

Two thin rods each lie along the x-axis from $x = 0$ to $x = L$. Rod P has linear mass density $\lambda(x) = \lambda_0 \dfrac{x}{L}$ and rod Q has $\lambda(x) = \lambda_0 \left(\dfrac{x}{L}\right)^2$.

(a) Indicate whether the center of mass of rod Q is farther from the origin than that of rod P, nearer to it, or at the same place. Justify your answer using qualitative reasoning beyond referencing equations.

(b) Derive an expression for the position of the center of mass of each rod, in terms of L. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

(c) Justify how your expressions in part (b) are or are not consistent with your reasoning in part (a).

(d) A third rod has $\lambda(x) = \lambda_0 \left(\dfrac{x}{L}\right)^n$. Indicate what happens to the position of its center of mass as n is made very large, and briefly justify your response.