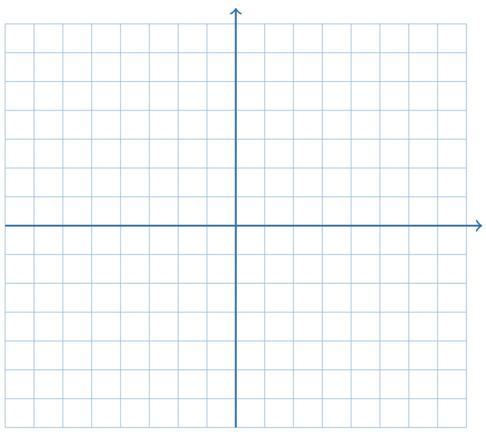
Problem Set 2: Force and Translational Dynamics

Name: ______________________________ **Date:** ______________

Topics 2.1 through 2.5. Every surface is frictionless unless otherwise specified. Take $\vec{a}_g = -9.80665\,\hat{\jmath}$ m/s^2. Out of 100 points.

1. Three small objects lie in the xy-plane: $m_1 = 2.00$ kg at the origin, $m_2 = 3.00$ kg at $(4.00$ m, $0)$, and $m_3 = 5.00$ kg at $(0,\ 6.00$ m$)$. **[17 pts]**



(a) Find the center of mass of the system, as a pair of coordinates. **[4 pts]**

(b) Plot the three objects and mark the center of mass on the grid, to a stated scale. **[4 pts]**

(c) A fourth object is added and the center of mass does not move. State everything this tells you about it, and everything it does not. **[4 pts]**

(d) A fourth object of mass 4.00 kg is instead placed so that the center of mass of all four lands exactly at the origin. Find where it must go, and explain why no choice of mass could put the center of mass outside the region spanned by the four objects. **[5 pts]**

2. A 6.00 kg block rests on a frictionless ramp inclined at $30.0°$ to the horizontal. A rope runs from the block up along the surface of the ramp to a post at the top, and the block does not move. **[17 pts]**

(a) Draw the free-body diagram of the block on the grid, and state which way your axes point and why. **[4 pts]**

(b) Find the tension in the rope and the magnitude of the normal force. **[4 pts]**

(c) A classmate says the normal force must be 58.8 N, since that is the weight. Say what is wrong with the reasoning and name the force that makes up the difference. **[4 pts]**

(d) The ramp angle is adjustable. Find the angle at which the tension and the normal force would be equal in magnitude, and say what each of them approaches as the ramp is steepened toward vertical. **[5 pts]**

3. Two blocks sit touching each other on a frictionless horizontal floor: block A has mass 4.00 kg and block B has mass 6.00 kg. A hand pushes horizontally on the outer face of A with a force of 30.0 N, and the two blocks move together. **[16 pts]**

(a) Treating the pair as one system, find the acceleration of its center of mass, then draw a separate free-body diagram for each block on the grid. **[4 pts]**

(b) Use B's diagram to find the force A exerts on B, and use A's diagram as an independent check. **[4 pts]**

(c) The same 30.0 N is applied to the outer face of B instead. Find the new contact force and explain why it changed although the acceleration of the system did not. **[4 pts]**

(d) A third block of mass 2.00 kg is placed in front of B and the original push on A is restored. Find the new acceleration and both contact forces, and state which of the three blocks now has the largest net force on it. **[4 pts]**

4. A 20.0 kg sign hangs at rest from two cables tied to a single point. The left cable runs up and to the left at $30.0°$ above the horizontal; the right cable runs up and to the right at $60.0°$ above the horizontal. **[17 pts]**

(a) Draw the free-body diagram of the sign on the grid. **[4 pts]**

(b) Write the two equilibrium equations and solve them for the two tensions. **[4 pts]**

(c) The two tensions add to more than the weight of the sign. Explain how that is consistent with equilibrium, and state which cable carries more by arguing from the geometry rather than the numbers. **[4 pts]**

(d) The right cable is replaced by a horizontal one tied to a wall, the left cable staying at $30.0°$. Find both new tensions, and state what has to be true of a cable arrangement for a tension to exceed the weight it holds by a factor of two or more. **[5 pts]**

5. A 3.00 kg block sits on a frictionless horizontal table. A light, inextensible string runs from it over an ideal pulley at the edge of the table to a 2.00 kg block hanging in the air. The system is released from rest. **[16 pts]**

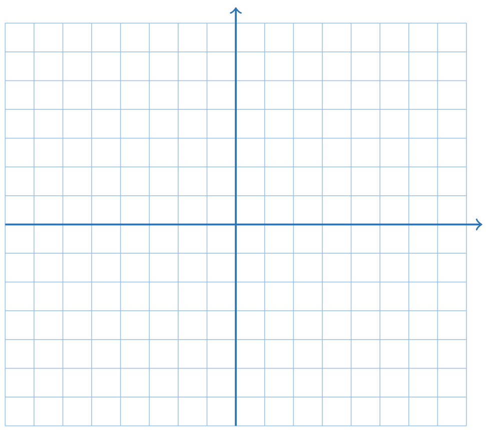
(a) Draw a free-body diagram for each block on the grid, stating the positive direction you have chosen for each, and explain why the two accelerations must have the same magnitude. **[4 pts]**

(b) Find the acceleration of the blocks and the tension in the string. **[4 pts]**

(c) A classmate writes that the tension equals the weight of the hanging block. Say what physical situation that answer would describe, and why it is not this one. **[4 pts]**

(d) The table block is replaced by one of unknown mass, and the hanging block is now measured to fall with an acceleration of exactly half the free-fall value. Find that mass, then state the largest acceleration this arrangement could ever produce and the mass that would give it. **[4 pts]**

6. A 5.00 kg cart runs along a straight frictionless track. Starting from rest, its speed rises steadily to 12.0 m/s over the first 4.00 s, holds at 12.0 m/s until $t = 7.00$ s, then falls steadily to rest at $t = 10.0$ s. **[17 pts]**



(a) Sketch the velocity against time on the grid, labeling both axes with quantity and unit. **[4 pts]**

(b) Find the net force on the cart in each of the three intervals, with signs, stating your positive direction, and name the interval in which the forces are balanced. **[4 pts]**

(c) A student riding the cart pushes off it during the middle interval. Explain why the velocity of the center of mass of the student-and-cart system does not change. **[4 pts]**

(d) The cart is instead brought from 12.0 m/s to rest in 1.00 s. Find the net force, say by what factor it exceeds the force in the third interval, and use that comparison to explain why a longer stop is gentler on whatever is being stopped. **[5 pts]**