Problem Set 1: Kinematics

Name: _____ **Date:** _____

Work symbolically wherever you can, and substitute once at the end. Units belong in every answer. Position is the vector $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$; resolve two-dimensional motion into components before you do anything else. Choose your positive directions, state them in writing, and keep them. Use $g = 10\ \text{m/s}^2$. Where a sketch is asked for, a grid sits at the left of the solution space with room to work beside it. Label the axes yourself and choose your own scale. This set is graded out of **100 points**; the point value of each question is marked beside its number. The **Calculus** part of each question is optional extra credit, up to **12 points**, and is not needed for full marks.

1. [18 pts]

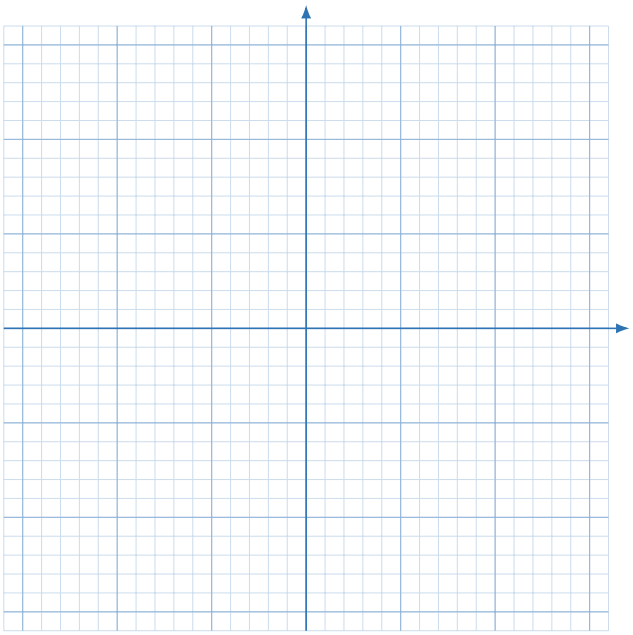
A drone's position changes from $\vec{r}_0 = (12\ \text{m})\hat{\imath} + (5\ \text{m})\hat{\jmath}$ to $\vec{r} = (-4\ \text{m})\hat{\imath} + (17\ \text{m})\hat{\jmath}$ over 8.0 s.

(a) Find $\Delta\vec{r}$ in component form.

(b) Find the magnitude of $\Delta\vec{r}$ and its direction, measured counterclockwise from the $+x$-axis.

(c) Find the average velocity $\vec{v}_{\text{avg}}$ in component form, and its magnitude.

(d) Plot $\vec{r}_0$, $\vec{r}$, and $\Delta\vec{r}$ on the grid, drawing $\Delta\vec{r}$ from the tip of $\vec{r}_0$ to the tip of $\vec{r}$.



Calculus (optional). Suppose the drone's position is $\vec{r}(t) = (12 - 2t)\hat{\imath} + (5 + 1.5t)\hat{\jmath}$, in meters with t in seconds. Differentiate to obtain $\vec{v}(t)$, confirm it agrees with your answer to (c), and say why the two agree here but would not for most motions. **[+2 pts]**

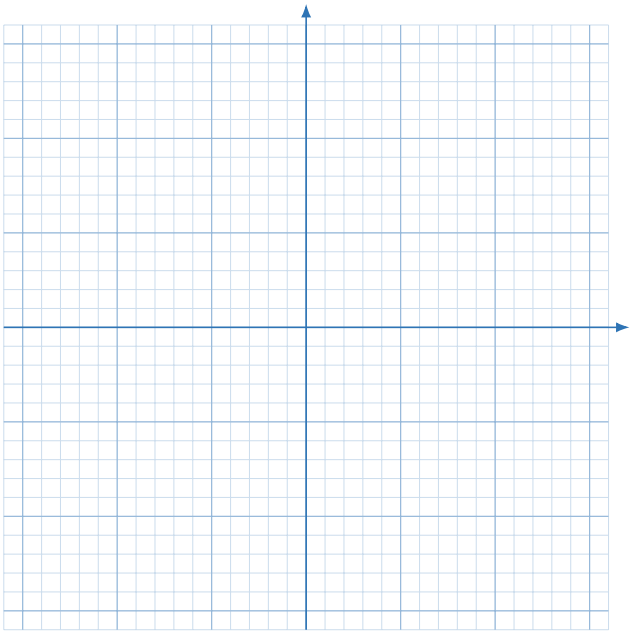
2. **[14 pts]**

A hiker walks 300 m due east, then turns and walks 400 m due north.

(a) Find the total distance walked.

(b) Find the magnitude of the displacement and its direction, measured north of east.

(c) Sketch both legs and the resultant displacement on the grid, to a stated scale.



(d) Explain why the answers to (a) and (b) differ, and state the one condition under which they would have been equal. Your answer should refer to the shape of the path, not to the numbers.

Calculus (optional). Distance traveled is $\int |\vec{v}\,|\,dt$. If the hiker walks the whole route at a steady 1.4 m/s, use that integral to find the total time for the trip. **[+2 pts]**

3. **[18 pts]**

A ball is launched from the origin with initial velocity $\vec{v}_0 = (15\ \text{m/s})\hat{\imath} + (20\ \text{m/s})\hat{\jmath}$. Its acceleration is $\vec{a} = (-10\ \text{m/s}^2)\hat{\jmath}$ throughout.

(a) Write $x(t)$ and $y(t)$, each as an explicit function of t.

(b) Find the time at which the ball reaches its greatest height, and explain which component of $\vec{v}$ told you.

(c) Find that greatest height.

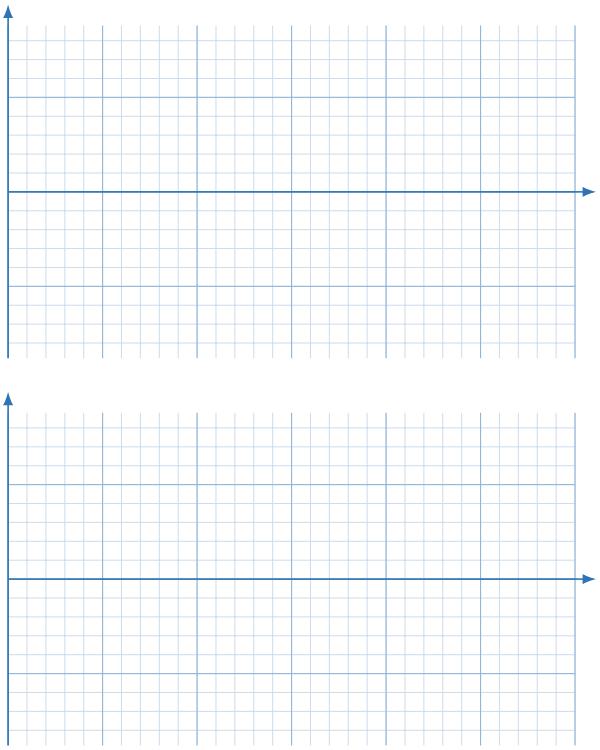
(d) Find the horizontal distance traveled when the ball returns to $y = 0$.

Calculus (optional). Obtain $v_y(t)$ and then $y(t)$ by integrating $a_y = -10\ \text{m/s}^2$ twice, and state what each constant of integration is physically. **[+2 pts]**

4. **[18 pts]**

The same launch as Problem 3.

(a) On the upper grid sketch v_x against t, and on the lower grid sketch v_y against t, both from $t = 0$ to $t = 4.0$ s. Label both axes with quantity and unit.



(b) Find the slope of the v_y graph, with units, and name the quantity it is.

(c) Find the area between the v_y curve and the time axis from $t = 0$ to $t = 2.0$ s, and say what physical quantity that area represents.

(d) The v_x graph is flat. State what that means about the horizontal motion, and explain why the ball still follows a curved path.

Calculus (optional). Show that the area you found in (c) is $\int_0^{2.0} v_y \, dt$, and evaluate that integral. **[+2 pts]**

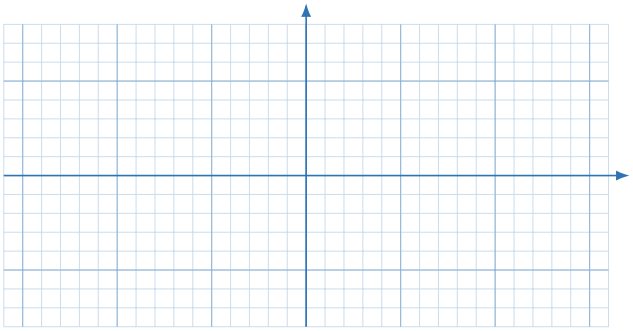
5. **[14 pts]**

A boat's engine drives it at 4.0 m/s relative to the water. The river current runs at 2.5 m/s relative to the ground. The boat heads directly upstream.

(a) Find the boat's velocity relative to the ground.

(b) A passenger walks from the stern toward the bow at 1.2 m/s relative to the boat. Find the passenger's velocity relative to the ground.

(c) Draw, to a stated scale, the boat's velocity relative to the water, the water's velocity relative to the ground, and their resultant.



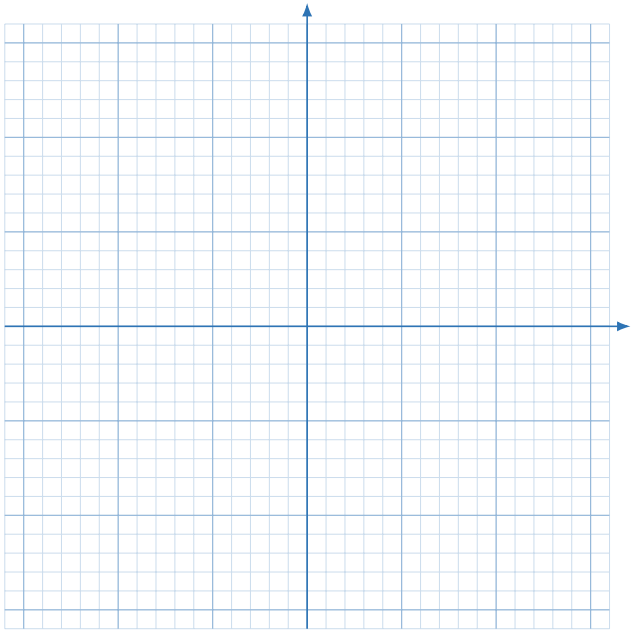
(d) Name one quantity here that the ground observer and the boat observer agree on, and one they do not. Justify each in a sentence.

Calculus (optional). The current is not steady: it runs at $v_c(t) = 2.5 + 0.3t$ m/s downstream. Find the boat's displacement relative to the ground over the first 10 s by integration, and interpret the result. **[+2 pts]**

6. **[18 pts]**

A projectile is launched from ground level at 25 m/s, at 37° above the horizontal.

(a) Resolve $\vec{v}_0$ into components, and sketch the launch vector with its components on the grid.



(b) Find the time to reach maximum height, and the maximum height.

(c) Find the total time of flight and the horizontal range.

(d) A classmate claims the projectile's acceleration is zero at the top of its path, "because it stops for an instant." Identify what is momentarily zero, state what the acceleration actually is there, and explain the difference.

Calculus (optional). The speed is $|\vec{v}\,| = \sqrt{v_x^2 + v_y^2}$. Differentiate to find the time at which the speed is least, and give that least speed. Compare with your answer to (b). **[+2 pts]**